Ancient Commentators on Aristotle

GENERAL EDITORS: RICHARD SORABJI AND MICHAEL GRIFFIN

IAMBLICHUS: On the General Science of Mathematics

Translated by
John Dillon & J.O. Urmson

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Iamblichus

On the General Science of Mathematics

Ancient Commentators on Aristotle

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Iamblichus

On the General Science of Mathematics

Translated by
John Dillon and J. O. Urmson

Introductory Essays by
Sebastian Gertz and Richard Sorabji

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Preface

Richard Sorabji

The late J. O. Urmson made a draft translation of this work of Iamblichus. On retirement from Corpus Christi College, Oxford, Urmson had offered to translate commentaries on Aristotle by Simplicius, and others, with the warning that his interest was only in Aristotle, not in the Neoplatonism of Simplicius: take it or leave it. I accepted with gratitude, and he accepted with grace, correction from peer reviewers on some Neoplatonist passages. He completed no fewer than six volumes of these translations during his retirement, with or without collaborators, but well before finishing, he had got intrigued by some of the Neoplatonists. He could not believe that a certain commentary on Aristotle's *On the Soul*, ascribed variously to Simplicius or to Priscian could possibly be by Simplicius, and asked to write an introduction, saying so. After his sixth volume, he quietly translated two other works of late Greek philosophy. One was written at the end of the fourth century by bishop Nemesius of Emesa in Syria, his *On the Nature of Man*, which has thrown light on Aristotle's greatest defender among his commentators, Alexander of Aphrodisias, and one was on the present work by Iamblichus. Urmson did not wish to write notes on these drafts, and he did not intend them for the series. But copies were made available in the library of the Institute of Classical Studies in London University and both were then put to use by other translators. The draft translation of Nemesius, *On the Nature of Man* was acknowledged in the first sentence of their preface by the subsequent translators, R. W. Sharples and P. J. van der Eijk.

After that, John Dillon, who has, with or without collaborators, translated four previous volumes for this series, and a very large proportion of Iamblichus' surviving work, then offered to revise Urmson's draft of the present work, to make a new version for this series. His translation was ready by 2018, when he asked me to arrange for annotation and introductions to be done by others.

The General Introduction has been written by Sebastian Gertz, and with some trepidation, I took on most of the annotation, along with a further essay, although conscious of how much more Dillon knew than I did about Iamblichus.

I was greatly helped by the scholarship on Iamblichus brought to my attention by D. S. Hutchinson and M. R. Johnson, including their study of Iamblichus' *Protrepticus*, 'Authenticating Aristotle's *Protrepticus*' (*Oxford Studies in Ancient Philosophy* 2005), and the results of their research reconstructing Aristotle's *Protrepticus*, published online at www.protrepticus.info, of which the most recent available version is from 2017. This incorporated text, translation, and commentary on Chapters 22, 23, 26, and 27 of Iamblichus' *De communi mathematica scientia* (*DCMS*), and in a subsequent collaborative exchange, Hutchinson sent me their unpublished research on Chapters 21, 24, and 25. He also provided comments on other parts of *DCMS*, notably Chapters 7 and 35. In further developments, Hutchinson and Johnson now argue that Chapters 21–27 of *DCMS* constitute another sequence of citations from the *Protrepticus* of Aristotle, a group of chapters that provides us with many new fragments of that lost work from more than six centuries earlier, and significantly increases our knowledge of Aristotle's lost dialogue.

Conventions

[. . .] Square brackets enclose words or phrases that have been added to the translation for purposes of clarity.

<. . .> Angle brackets enclose conjectures relating to the Greek text, i.e. additions to the transmitted text deriving from parallel sources and editorial conjecture, and transposition of words or phrases. Accompanying notes provide further details.

(. . .) Round brackets, besides being used for ordinary parentheses, contain transliterated Greek words.

Abbreviations

Throughout the notes the following abbreviations have been used: SG, Sebastian Gertz; RRKS, Richard Sorabji; From DSH or From DSH/MRJ, as reported with permission from the research of Doug Hutchinson and Monte Johnson. All other notes are by John Dillon, marked JMD only in cases where there are additions by others to his footnote.

DCMS	*De communi mathematica Scientia* (the work translated in this volume)
DK	Diels, H., and W. Kranz, *Die Fragmente der Vorsokratiker*, 3 vols (Berlin: Weidmann, 1952)
in DA	*Commentary on Aristotle's De Anima*
in Int.	*Commentary on Aristotle's On Interpretation*
in Metaph.	*Commentary on Aristotle's Metaphysics*
OSAP	*Oxford Studies in Ancient Philosophy*
PA	*Parts of Animals* (Aristotle)
Rep.	*Republic* (Plato)

Introduction

Sebastian Gertz

The general context of the work

The treatise *On the General Science of Mathematics*[1] comes as the third in a set of ten treatises, composed by the philosopher Iamblichus (*c.* 245–325 CE) in a semi-popular mode – probably primarily for the use of the students in his school in Apamea – on the nature and scope of the Pythagorean tradition in philosophy.[2] In the two treatises preceding this one, the *Peri tou Pythagorikou biou,* or *On the Pythagoric Way of Life,* and the *Protreptikos,* Iamblichus has set out the general principles of Pythagorean philosophy, together with a good deal of biographic and hortatory material, and he now proposes to turn more particularly to the place of mathematics in the system. He emphasizes the close connection of mathematics with the parts of philosophy, namely ethics, physics, and theology and lays out a hierarchy of the subdisciplines of mathematics, with arithmetic, geometry, harmonics and astronomy ranked in order of priority.

Within the context of Iamblichus' exposition of Pythagoreanism, *DCMS* occupies something of a transitional place: while emphasizing the practical relevance of mathematics for the pursuit of the good life, it sets out the metaphysical and conceptual basis for the subsequent books of the series, which deal with more specific aspects of Pythagorean mathematics.

Of these later books, only the fourth title survives in full, a series of reflections (traditionally described as a 'commentary') on the *Introduction to Arithmetic* by the Neopythagorean philosopher and mathematician Nicomachus of Gerasa, about whose biography very little is securely known (he is likely to have been active in the first half of the second century CE).[3] Books 5–7 of Iamblichus' *On Pythagoreanism* would have discussed the application of arithmetic to the three parts of philosophy, i.e. physics, ethics,

and theology. The three works survive only in fragments: Michael Psellus, a Byzantine scholar (eleventh century CE), excerpted portions of them, which he published under his own name, probably with only some minor additions of his own. It has been estimated that about one twenty-fifth of Book 5 (*On Arithmetic in Physical Matters*), and about one fiftieth of Books 6 (*On Arithmetic in Ethical Matters*) and 7 (*On Arithmetic in Theological Matters*) each, have made it into Psellus' two *opuscula On Physical Number* and *On Ethical and Theological Number* respectively.[4] Of Books 8–9, only the bare titles remain (*On Pythagorean Geometry* and *On Pythagorean Music*); most likely, Iamblichus' original design would also have included a tenth book *On Pythagorean Astronomy*, so as to cover all the four parts of mathematics.

The subject matter of *DCMS*

As a text about mathematics, Iamblichus' *DCMS* is rather like a cookbook without recipes: there is, in fact, very little mathematics contained in its pages, the main focus of the work being metaphysical and epistemological. Perhaps the clearest formulation of what the study of the 'common science of mathematics' entails can be found in the following passage:

> Thus the subject matters of the study of mathematics which extend in common throughout the whole of this science are those theorems such as are common to all, capable of being adapted to numbers, to magnitudes and also to harmonics, astronomy and all its other parts.
>
> (18,24–19,1)

Iamblichus goes on to list at some length the kinds of theorems that are common to all the individual mathematical sciences, such as theorems about ratios, or 'concerning combinations and divisions in general'; but here as elsewhere, his interest is less in specifics than it is with providing 'a synoptic view of the interrelationship of all mathematical sciences and the nature of being', as he puts it (22,6–8).

It is worth emphasizing at this point that the subject matter of the *DMCS*, 'the common science of mathematics', is not primarily the kind of mathematics as it would have been practised in Iamblichus' time, much of which he would have dismissed as 'logic-chopping and divorced from the true nature of the

facts' (62,5–6). Pythagorean mathematics, as understood by Iamblichus, is something of a normative ideal, a method of intellectual enquiry that is strictly subordinated to the two goals of understanding reality and shaping one's own character in imitation of the harmonious order that underlies mathematical structures. It is 'Pythagorean', not in the sense that it agrees or seeks to agree with a securely established body of teachings, but rather in the sense of aiming to reconstitute a lost body of knowledge that survives at most in myths and fragments. Iamblichus himself provides a clear statement of the methodology behind his project in the *DCMS*:

> We must start from small clues and continually build them up and increase them; we must take them up to suitable starting points, interpolate what is missing, and speculate so far as possible as to their doctrinal positions, what they would say if it were possible for one of them to inform us. But as it is, from pursuing the logical consequences, we are able to uncover their teachings suitably from what they have indisputably handed down to us. For such methods of investigation will enable us either to arrive at the actual Pythagorean mathematical science or to come very close to it, to the highest possible degree. (68,11–15)

The structure and argument of *DCMS*

Throughout its thirty-five chapters, *DCMS* covers a wide range of topics, ranging from the first principles of mathematics to the etymology of that science. Although it is hazardous to force the entire work under a single line of argument, one may reasonably take the following summary of its key postulates as the guiding principle that provides some kind of unity to the entire composition:

> Let the following postulates be assumed in general for the totality of the objects of mathematics: that their mode of existence is incorporeal and self-subsistent, holding an intermediate rank between indivisible beings and those that are divided about bodies, both as forms and as reason-principles, being assigned the middle status between the simple and the divisible, being purer than the latter, more variegated than the former; that they make use of synthesis and division, but oversee the synthesized and the divided without coming to be and eternally; they are inferior to intelligible beings but prior

> to natural ones; they are in beauty, in order, and in exactitude superior to things visible but inferior to the intelligible, and similarly they have an intermediate symmetry and compatibility; they have the power to transport and lead over to the indivisible forms, since they are akin to them, and they lead away from corporeal concerns those who have become accustomed to them, converting them to the divine beings, as if on a ladder leading up to the heights.
>
> (10,7–24)

Described in this passage we find two of the core themes that recur throughout *DCMS*: first, the position of mathematical objects as intermediate between physical objects and intelligible forms; and second, the power of mathematics to accustom the mind to thinking about immaterial objects, acting as a ladder that can lead the committed student to the higher reaches of philosophy, in particular metaphysics, and dishabituate him or her from exclusive concern with the objects of sense perception.

The topics and contents of the individual chapters can be gleaned from the short summaries that precede each and that may well go back to Iamblichus himself. A list of these summaries, included in this translation (see pp. 33–6), would have prefaced the entire work. Grouping the chapters together thematically results in the following outline:[5]

Introduction
Chapter 1: Preface
Chapter 2: Synopsis of the common theory of mathematics

Principles
Chapter 3: The principles of mathematics as a whole
Chapter 4: The principles of each branch of mathematics

The common theory
Chapter 5: The common features of all mathematical sciences
Chapter 6: Goal of the study of mathematics
Chapter 7: The particular subject of each branch of mathematics
Chapter 8: The faculty of judgement (*kriterion*) appropriate to mathematics
Chapters 9–10: Soul and mathematics
Chapter 11: How knowledge of mathematics is acquired by personal discovery or instruction

Chapter 12: Preliminary account of the capacities (*dunameis*) of mathematics
Chapter 13: Elements and kinds (*genê*) of mathematics
Chapter 14: Similarity and dissimilarity in mathematics

Application of mathematics to other areas (1)
Chapter 15: How mathematics extends to all of philosophy
Chapter 16: Mathematics applied to arts and crafts

Methodological reflections (1)
Chapter 17: The order of presentation of the mathematical sciences
Chapter 18: Pythagorean methods of teaching mathematics
Chapter 19: The division of mathematics according to the Pythagoreans
Chapter 20: Method of division in mathematics

Historical survey
Chapter 21: Predecessors of Pythagoras
Chapters 22–25: Pythagorean mathematics

Digression on the utility of mathematics
Chapter 26: Defence of the utility of mathematics

Methodological reflections (2)
Chapter 27: What a well-educated person can expect of a mathematician
Chapter 28: What characterizes a mathematical line of argument (*logos*)?
Chapter 29: How does mathematics relate to dialectics?

Application of mathematics to other areas (2)
Chapter 30: The contribution of mathematics to other parts of philosophy
Chapter 31: How the Pythagoreans applied mathematics to other areas of philosophy
Chapter 32: How mathematics can be applied to the objects of sense perception

Conclusion
Chapter 33: Concluding account of the common genus of mathematics
Chapter 34: Etymology of the name 'mathematics'
Chapter 35: Summary

Iamblichus' use of quotation

There can be little doubt that Iamblichus must have had access to a well-equipped library; throughout the *DCMS*, he borrows liberally from a variety of Neopythagorean and other philosophical works, at times directly quoting his sources and at times excerpting or paraphrasing other authors without acknowledgement. In places, we find Iamblichus or his source weaving together complex tapestries of Platonic themes without drawing attention to the provenance of his borrowings; Chapter 6, for example, contains an extensive paraphrase of *Epinomis* 991B-92B as well as several thinly disguised extracts from Book 7 of the *Republic*.[6] Elsewhere, Iamblichus can be seen to borrow long stretches of text from Aristotle; two notable examples are (1) Chapter 26 of the *DCMS*, where he defends the value of abstract mathematics against its supposed uselessness, in a passage that ultimately derives from Aristotle's work *Protrepticus* (available to us only in fragments); and (2) Chapter 27, whose opening section is lifted either directly from Aristotle's *Parts of Animals* or, once more, from Aristotle's *Protrepticus*.[7] Such a process of stitching together excerpts from a variety of sources at times obscures the train of thought behind the overall composition of *DCMS* and may explain a number of awkward transitions and apparent gaps in Iamblichus' argument. While it may be a step too far to say that the *DCMS* is 'quite obviously a series of excerpts from different authors rather than Iamblichus' own work', as Merlan puts it, it is worth bearing in mind the complex process of composition that lies behind it.[8]

A case in point is the difficult question of Iamblichus' thoughts on the relationship between mathematics and the soul. In Chapter 4 of *DCMS*, he is careful to insist that mathematical entities and the principles of life, which include the soul, should be thought of as belonging to different spheres of existence, in so far as the former do not act as motive causes, while the latter certainly do. Elsewhere, however, this distinction is much less clearly drawn: in Chapter 9, for example, he almost identifies the soul with mathematics, arguing that 'we conceive of the soul as having its being in the common reason-principles of all branches of mathematics' (41,25). Scholars have long suspected that less than careful editing of his sources on Iamblichus' part is to blame for the apparent tension between these two chapters, one maintaining the identity of the soul with mathematics, the other denying it. In particular, it is likely that

Chapter 4 in fact contains an extract from the philosopher Speusippus (active in the fourth century BCE), about whose views we are otherwise informed through Aristotle.[9] It is Speusippus, and not Iamblichus, who would have placed the objects of mathematics at a level of reality different from that of soul and to whom many of the more peculiar doctrines of Chapter 4 (such as the view that the One is neither beautiful nor good) should probably be attributed.[10]

Iamblichus' use of pseudo-epigraphic late Neopythagoreans

Another noteworthy aspect of Iamblichus' handling of his sources is his presentation of Plato, whose conception of mathematics profoundly influenced the Neopythagorean authors quoted or otherwise incorporated in the *DCMS*. Curiously, instead of directly quoting from Plato, Iamblichus defers in several places to the spurious authority of pseudo-epigraphic Neopythagorean writers such as 'Brotinus' or 'Archytas', who themselves borrowed from, if not outright plagiarized, Plato's works. A striking example of this phenomenon can be found in Chapter 8, which quotes a lengthy passage from a treatise *On Intellect and Sensation* by 'Archytas'. In the extract, 'Archytas' distinguishes four modes of cognition (intellect, scientific knowledge, opinion, and sensation) and argues that a different class of knowable object corresponds to each mode. To illustrate his point, he reproduces without acknowledgement the famous simile of the divided line from Book 7 of Plato's *Republic*, which likens the four levels of reality to a line cut into two unequal sections, each of which is cut into two further unequal subsections. When expounding the meaning of this simile, Iamblichus does not remark on the obvious Platonic borrowings of his source 'Archytas' at all, perhaps out of commitment to the view that Plato in some way derived his philosophy from more ancient, Pythagorean wisdom.

Influence

The profound influence of Iamblichus' Pythagoreanizing conception of mathematics on later ancient philosophy can be illustrated by the fact that two major Platonist thinkers active during the fifth century CE, Syrianus and

Proclus of Athens, had first-hand knowledge of the *DCMS*. In his *Commentary on Aristotle's Metaphysics*, Syrianus critiques at length what he took to be Aristotle's position on the nature of mathematical entities, namely that they do not have independent existence but are instead derived from the objects of sense-perception through a process of abstraction (*aphairesis*). At the end of his discussion, Syrianus offers his readers some guidance as to where a true account of mathematics is to be found:

> But if one were to acquaint himself with the genuine doctrines of the Pythagoreans, he will come to learn clearly also the essence and the potency and the actualities of mathematical science as a whole, what is the body of theory common to all of it, and into how many sub-classes it extends; what are the first principles of all the various science and in what way they differ from other principles, such as are the first principles of other beings, and how such principles provide a common mode of causation (*aitia*) in respect of all the mathematical sciences.
>
> (101,26–101,32)[11]

In the above passage and its continuation, Syrianus summarizes more or less sequentially the main topics of Iamblichus' *DCMS* and credits him with having enriched the basic teachings of Pythagorean mathematics with 'constructive reasonings, proofs, and more intellectual theorizings'. It is clear, therefore, that Syrianus was directly familiar with the *DCMS* and saw in it an authoritative and truthful account of the nature and status of mathematical science and the objects of its study.

Syrianus' star pupil and eventual successor at the Platonic Academy in Athens, Proclus, likewise owes a debt to Iamblichus' treatise. In the first prologue to his *Commentary on Euclid's Elements*, Proclus expounds his general philosophy of mathematics and in the process covers much the same ground as the *DCMS*. Like Iamblichus before him, Proclus discusses the intermediary status of mathematical objects, their derivation from the principles of the Limit and the Unlimited and the cognitive faculties appropriate to them, while defending the utility of mathematics against its detractors. Unlike Iamblichus, however, Proclus makes only passing reference to the Pythagoreans, preferring instead to allude to and expound Platonic source texts, such as *Republic* 509D-511E, *Timaeus* 35A-36C, and *Meno* 82B-85C. Overall, Proclus' exposition of the general science of mathematics greatly clarifies the thought of Iamblichus'

DCMS, from which it draws inspiration: where Iamblichus presents two potentially conflicting accounts of the relation between the soul and mathematics, for example, Proclus sets forth a single coherent view, according to which the soul is composed of mathematics.

From a wider philosophical vantage point, we can credit Iamblichus with articulating an alternative to Aristotle's theory that mathematical knowledge is the result of a process of abstraction from particulars. On Iamblichus' rival account, mathematical objects are images or copies of the intelligible forms. How precisely these copies or images are produced by the soul is left somewhat unclear; it fell to Syrianus and Proclus to flesh out the embryonic theory bequeathed to them by Iamblichus and to bolster it with supporting arguments. The resulting account, christened with the ungainly term 'projectionism', maintains that mathematics in its highest, Pythagorean form is the result of the soul projecting its own internal reason principles onto the imagination (*phantasia*).[12] In this way, Plato's celebrated doctrine that all a priori knowledge, including the truths of mathematics, is acquired by recollecting what was once known but had been forgotten at birth survives, *mutatis mutandis*, until the last days of the Platonic Academy in Athens.

Notes

1 *Peri tés koinés mathematikés epistémés,* commonly referred to by its Latin equivalent *De communi mathematica scientia* (*DCMS*). For convenience, it will generally henceforth be referred to by its abbreviation.

2 Cf. D. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity* (Oxford: Clarendon Press, 1990), p. 32.

3 On Nicomachus' philosophy, see J. Dillon, *The Middle Platonists* (Ithaca: Cornell University Press, 1996), pp. 352–60.

4 Cf. D. O'Meara, *Pythagoras Revived*, p. 59.

5 I am here adapting Gerald Bechtle's resumé of the *DCMS* on pp. 18–21 of his 2005 habilitation, 'Philosophie et mathématique pythagorisante chez Jamblique'.

6 See n. 60 of the translation for a close parallel between Iamblichus' arrangement of passages from *Epinomis* and the *Republic* and that found in an earlier source (Nicomachus of Gerasa). The same note also contains a detailed list of correspondences between Chapter 6 of *DCMS* and the relevant passages in Plato. See also n. 11 by Richard Sorabji on Iamblichus' use of *paradeixis*.

7 On Chapters 26 and 27, see D. Hutchinson and M. R. Johnson's commentary, available online at http://www.protrepticus.info/26comm.pdf and http://www.protrepticus.info/27comm.pdf respectively (last accessed in May 2019). For further discussion and references, see also the notes *ad loc*. in this translation, especially n. 186 (on the relationship between Chapter 26 and Aristotle's *Protrepticus*) and n. 205 (on Chapter 27 and Aristotle's *On Part of Animals* or *Protrepticus*).

8 P. Merlan, *From Platonism to Neoplatonism* (The Hague: Martinus Nijhoff, 1975), p. 28.

9 See especially *Metaphysics* 1092a11–22.

10 On this issue, see Merlan, *From Platonism to Neoplatonism*, pp. 15–32; 96–140 and J. Dillon, 'Speusippus in Iamblichus', *Phronesis* 29 (1984), 325–32 for a defence of Merlan's position.

11 Translated by John Dillon and Dominic O'Meara, *Syrianus: On Aristotle Metaphysics 13–14* (London: Duckworth, 2006).

12 For an in-depth survey of the debate between abstractionism and projectionism in the Neoplatonic commentators, see I. Mueller, 'Aristotle's doctrine of abstraction in the commentators', in R. Sorabji, *Aristotle Transformed* (London: Duckworth, 1990), pp. 501–20.

Iamblichus' Projects and Genres

Richard Sorabji

Iamblichus had a variety of projects and genres, and I shall come last to his *On the General Science of Mathematics* (*De communi mathematica scientia*, or *DCMS*) and to his Pythagorean sequence of books to which it belongs, in order to bring out how it differs from other projects.

Iamblichus' school at Apamea

Iamblichus lived in Syria from about 245 to the early 320s CE and founded a school there in a beautiful site at Apamea. Dillon[1] points out that his name is not Greek, but Semitic, originally *ya-mliku*, and that an ancestor was Sampsigeramos, founder of the line of priest-kings of Emesa in Syria. This may be relevant to his haughty defence of his religious views.

He was revered as a priest and miracle worker by his students, according to Eunapius, *Lives of the Sophists*,[2] and performed animal sacrifices at dawn, the very opposite of what was advocated by his one-time philosophical teacher Porphyry, in a series of works, including Book 2 of his *On Abstinence from Killing Animals*, as right for a philosopher.[3] Porphyry thought the immaterial gods required the sacrifice only of a pure mind, whereas animal sacrifice merely attracted bad demons.[4] If Porphyry's earlier fragmentary *Philosophy from Oracles* describes details of sacrificial practice, it does not follow that Porphyry approved them even then.

I have offered an interpretation, at the invitation of the archaeologists Jean and Janine Balty, of the mosaics excavated at the site of Iamblichus' school,[5] and documented by Janine Balty, mercifully before the recent war which may have caused serious damage. One mosaic clearly belonged to the philosophers, since it represents Socrates, who is named, surrounded by his interlocutors.

The bigger one found next to it I interpreted as the mosaic of the rhetoricians and as suggesting a very different message, 'Rhetoric will show you how to argue the opposite of anything'.[6] Iamblichus himself was trained in rhetoric and could well have given classes there too. The presence of rhetoric would have ensured a large clientele of students and is relevant to the survival of the premises after Iamblichus' death in 325 long enough for the Emperor Julian, who ruled in Constantinople from 361 to 363, to install in the school in honour of Iamblichus' memory the mosaics still viewable at the time of my visit.

Iamblichus *versus* Porphyry on priestly power

Iamblichus' surviving philosophical writings are of very diverse kinds. Many are lost, and the only one surviving in full concerns a major controversy on priestly practice with his one-time teacher Porphyry. I take it first in order to contrast it with *On the General Science of Mathematics* and the incomplete sequence of Pythagorean books to which that belongs. Iamblichus' complete book on priestly practice was grandly entitled, 'The Reply of Master Abammon to the Letter of Porphyry to Anebo', although it now tends to be known under Marsilio Ficino's blander title as *On the Mysteries of the Egyptians*.

Iamblichus took theurgical practice to be able to provide union with God. The Christian Augustine, in his great work *The City of God* (10.9), reports on Porphyry's quite different treatment of theurgy in his (Porphyry's) lost work, *Return of the Soul*, and says that there Porphyry took theurgy to purify only the imaginative part of the soul so that it could help us to see visions, but not the intellectual part, so that it cannot lead us back to God. Augustine expresses respect for Porphyry as Christianity's most formidable critic.

Porphyry's *Letter to Anebo* had addressed questions to Iamblichus about Egyptian priestly power as if to an Egyptian priest of that name. Porphyry's letter, surviving now only in fragments, may have expressed a respectful, even if critical, curiosity, if we may judge from Augustine's response to Porphyry from his different Christian point of view, and from Porphyry's having dedicated a book, now lost, to Iamblichus, *Know Thyself*, about philosophical practice. Iamblichus himself claimed to have 'heard' Porphyry, either listening

to him speak as a senior member of Plotinus' seminar in Rome, or else visiting him as Plotinus' subsequent successor there, and even more important, to have once devoted (*prostheis*) himself to Porphyry. But Iamblichus' detailed reply to Porphyry's letter,[7] purports by an act of one-upmanship to come from Anebo's teacher, whose name, Abammon, may have meant in Egyptian 'Father of God'. The title Abammon had been reserved by Porphyry for someone who has reached the highest level of virtue he recognized.

In his *On the Mysteries of the Egyptians*, Iamblichus says that only theurgic priests know how to name the gods and how to use in ritual these names, which are divine symbolic character marks, a claim that was to be repeated in Athens by Proclus. And the gods are said to have revealed that the language of the sacred races such as Syrians (his own race) and Egyptians provides the most suitable names. Theurgic union with the gods is achieved by ineffable actions and the power of symbols which are thought only by the gods and could not be achieved by the intellectual thought of philosophy, which is thereby classed as inferior.[8]

Iamblichus' commentaries on Plato and Aristotle and his philosophical skill and influence

One of Porphyry's questions gets an answer which reveals Iamblichus' inspired use of philosophical history, in this case neglected history. Porphyry asks why ritual practices include erotic festivals. Iamblichus' reply takes up obscene sights and sounds in religious rituals, the setting up of phalli and the utterance of obscenities, and he offers two justifications. After his first suggestion, that this may work on people as an aversion therapy, he makes the fullest surviving reference to Aristotle's theory of the cathartic function of tragedy and comedy in the theatre since Aristotle produced it 650 years earlier. A moderate exercise of our emotions may make them moderate, as Aristotle wished, and then even rid us of them.[9] Iamblichus here makes use of a neglected idea in the *history* of philosophy, in order to answer a *philosophical* challenge to his priestly beliefs.

Fragments survive of Iamblichus' own seven commentaries on Plato and five on Aristotle. For Plato's dialogues, he established a respected curriculum of

selected dialogues in a set order.[10] The commentaries on Aristotle covered Aristotle's *Categories*, *On Interpretation*, *Prior Analytics*, *On the Heavens*, and *On the Soul*.

The first of the heads of the Athenian school known to have been influenced, Syrianus, head from 432 to 437 CE, closely copied the headings of Iamblichus' *DCMS* Chapters from 2 to 32 in his commentary on Aristotle's *Metaphysics* (101,26–102,35),[11] and Syrianus' commentary is influenced by Iamblichus throughout. The next school head, Syrianus' more famous pupil, Proclus, preserves more than anyone of Iamblichus' ideas.[12] Iamblichus' commentary on Aristotle's *Categories* was extensively preserved in the commentary of the Athenian Simplicius[13] on Aristotle's same work. Hermeias, visiting Athens from Alexandria, preserves a lot of Iamblichus in his commentary on Plato's *Phaedrus*,[14] based on the lectures of Syrianus in Athens, and will have brought these ideas back to Alexandria. Other Athenians also quoted Iamblichus.[15]

Hermeias' son Ammonius, head of the Alexandrian school in the fifth to sixth centuries was to acknowledge Iamblichus for one of his cleverest ideas in his own commentary on Aristotle's *On Interpretation*.[16] Knowledge, Iamblichus had said, need not have the same status as the known. Thus in the gods there can be timeless knowledge of the temporal, changeless knowledge of change, and determinate knowledge of the indeterminate. Ammonius makes use of Iamblichus' idea by applying it to the gods' timeless knowledge of the temporal *future*. His anxiety, like Aristotle's in the passage commented on, is about whether our future actions are already inevitable and he is also interested in divine providential help, although in both cases his view is only adumbrated. The gods surely know our future doings, but does this mean that we cannot do otherwise, since they have *already* foreknown, as well as known *infallibly*, what we will do, so that it is too late to decide otherwise? Not *already*, for the gods have no position in time, so their knowledge is timeless, and should not be described as holding 'already'. Hence we are not too late, and there is another surprising consequence. Our prayers to the gods can succeed not because they tell the gods that we need their assistance in the immediate future. The gods are outside past, present, and future, and their response to prayer depends on timeless knowledge and activity.

Julian, Emperor 361–2, and his revival of Iamblichus' theological tradition

After Iamblichus' death, his successor, Aidesius continued the school elsewhere in Pergamum. The Emperor Julian, who honoured Iamblichus posthumously with the installation of the mosaics back in Pergamum, did not persuade Aidesius to mentor him, so brought Aidesius' pupil Maximus to Constantinople. This was part of Julian's project, frustrated by his early death and short reign, to turn the empire back from Christianity to the pagan gods. He offered a prize for a submission on Aristotle's logic, and awarded it to Maximus, instead of to the great commentator on Aristotle, Themistius, whose objections to Maximus' submission have just been edited from a surviving Arabic version.[17] Julian also wrote to Aidesius' other pupil Priscus in Athens, for copies of Iamblichus' work and obtained Iamblichus' now lost *Commentary* on the theological poem, *The Chaldaean Oracles*.[18]

Iamblichus' priestly influence in Athens more than in Alexandria

The Athenian Damascius was to confirm the effect in Athens of Iamblichus' celebration of priestly power, when he said that the first Neoplatonists, Plotinus and Porphyry, had preferred philosophy, but Iamblichus, and the Athenians Syrianus and Proclus preferred theurgy.[19] Nonetheless, in the other major Neoplatonist school at Alexandria, there were fewer quotations from Iamblichus after Hermeias.[20] In Alexandria from Ammonius' headship onwards, support for Iamblichus' priestly ideas was modified. Ammonius made a deal with the Alexandrian city fathers, who had paid for his training, to head the teaching of Greek philosophy after the disruption of riots that started between Christian and pagan students and the subsequent persecution of pagan philosophy teachers in 486–9. The troubles had arisen out of a dispute on the ability of Egyptian priests of Isis to cure a Neoplatonist's wife of infertility. I have taken the deal to involve merely the agreement not to parade the claims of priestly ritual. But I take Ammonius, like Porphyry before him, not to have attached the same importance to priestly power and all he would

have needed to do was to confine the syllabus to Aristotle and to parts of Plato that would not require priestly power to be discussed.[21] For example, in his commentary on Aristotle's *On Interpretation*, a work which discusses names, he starts by saying (1,6–11), that he has recorded the interpretations of his divine teacher, Proclus. It has been shown by Anne Sheppard that he closely reflects his Athenian teacher Proclus' commentary also on *Plato's* main work on names and whether or not they are natural, Plato's *Cratylus*.[22] However, he omits reference to that *Cratylus* commentary in so far as it contains pages and pages on the naturalness of divine names and their utility to theurgy and he writes no commentary himself on that work of Plato's. Instead, he sticks to Aristotle's *On Interpretation* and to expounding Aristotle's view that names are due to conventional agreement (*sunthêkê*, *On Interpretation* 16a19) among humans, with which he appears to agree, and he remains silent on Proclus' contrary beliefs.[23] Ammonius uses not only Aristotle's word *sunthêkê*, but also the word Porphyry had used in agreeing with Aristotle, that names are something produced by human *imposition* (*thesis*) of them.

A little later in Alexandria, Ammonius' next successor but one in the sixth century CE, Olympiodorus, in the changed circumstance of a classroom not mixed, but consisting almost wholly of Christian students, is able to tell them of his belief in theurgy, but as something *unfamiliar*, and he has weakened the concept. Instead of giving one union with God, theurgy gives one union with the objects of intellect, while philosophy gives one identity with intellect itself.

Mathematics and the Pythagorean sequence: History retold by edited quotation

DCMS is the third of Iamblichus' Pythagorean sequence of ten books, of which the last six are lost. The first is a work on the Pythagorean lifestyle, *On the Pythagorean* (*Way of*) *Life*.[24] The second is the *Protrepticus* or *Exhortation to Philosophy.*[25] The third is the present text, *DCMS*, and the fourth is an expanded reiteration of Nicomachus of Gerasa's *Introduction to Arithmetic*. Dominic O'Meara provides an introduction to the series in his *Pythagoras Revived*.[26]

The suggestion that the whole of Iamblichus' *DCMS* might be quotation was already made by Philip Merlan in 1960, and repeated by John Dillon in

1984, with the further suggestion in 1987 that this might be true of Iamblichus' whole Pythagorean sequence of books. Doug Hutchinson and Monte Johnson had been illustrating the point about quotation in publications since 2005, starting with a special interest in quotation from Aristotle's fragmentary *Protrepticus*, or *Exhortation to Philosophy*, which is also the title of Iamblichus' second book in his Pythagorean sequence.[27]

After the publication in 2005 of their study of the citation techniques of Iamblichus in his *Protrepticus* and the implications for authenticating Aristotle's *Protrepticus*, Hutchinson and Johnson proceeded to reconstruct the lost work in provisional booklet formats and made these available on their website. The 2017 version at www.protrepticus.info/protr2017x20.pdf included quotations (or paraphrases) from both the *Exhortation* and the *DCMS* of Iamblichus. They also there reconstructed Aristotle's *Exhortation* as a fictional dialogue between three contemporaries, Isocrates of Athens, a philosophy teacher hostile to mathematics as useless, Heraclides of Pontus, Aristotle's fellow student in Plato's Academy and enthusiastic supporter of Pythagoreanism, and Aristotle himself, giving his own views at the early stage of his apprenticeship with Plato. On this reconstruction, quotations from Aristotle's *Exhortation* may represent any of the three competing views.

The case for Iamblichus quoting from Aristotle's *Exhortation* was made first by Ingram Bywater in 1869, and passages in *DCMS* Chapter 26 were included by Rose in 1886 in his collection of fragments of Aristotle's lost works.[28] But in 'A new fragment of Aristotle' in Chapter 6 in his *Platonism and Neoplatonism*, 2nd edn, 1960, Philip Merlan added Chapter 23 of Iamblichus' *DCMS*, saying (p. 144) that *DCMS* Chapter 23 is 'either a series of quotations or one quotation from Aristotle'. He too saw the quotations as being drawn from Aristotle's early *Exhortation to Philosophy* (*Protrepticus*), in which Aristotle was still under the spell of his teacher Plato and Plato's Academy in their estimation of the status of mathematical objects. In 2017, Hutchinson and Johnson have seen *DCMS* Chapters 21–7 as drawn from Aristotle's *Exhortation*, with two examples of text re-use: the same passages are cited in both *DCMS* 26 and *Protrepticus* 6, and the same passages are cited in both *DCMS* 25 and *On the Pythagorean Way of Life* 18.

Merlan had gone so far as to say that 'All of [*DCMS*] consists of excerpts and was not meant to be taken as Iamblichus' own work in our usual sense of

the word' (p. 158). In another chapter of the second edition of his book, Chapter 5, Merlan argued that Chapter 4 of *DCMS* was extensively quoting Aristotle's fellow student Speusippus. John Dillon agreed,[29] with the qualification that the end of *DCMS* Chapter 4 was summary, rather than quotation, of its source, and Dillon took it that in most of the rest of *DCMS* Iamblichus seemed to be quoting a sequence of sources virtually verbatim. In 1987 Dillon applied this conclusion to Iamblichus' series of Pythagorean works as a whole, that it was largely 'centos of passages from earlier writers, including Plato and Aristotle. . . . All Iamblichus contributes is introductory and bridge material'.[30]

This conclusion was argued to be correct, at least for the case of *Protrepticus*, in Hutchinson's and Johnson's 2005 *OSAP* article, in which they studied the Plato citations in comparison with the originals and concluded that

> the method of literary construction used by Iamblichus for both the Plato and the Aristotle chapters is to assemble into each chapter one or a few pure blocks of quotation in a natural sequence, marking them off from each other by thin comments of his own. The blocks of quotation are solid and pure, in that no passages are removed from and no extraneous passages are added to them, though they are sometimes carefully modified. The sequence of quotation blocks is natural, in that Iamblichus never returns to a work he has earlier been quoting from, nor does he ever return to any earlier part of a work he has been quoting from.
>
> (p. 194)

In the 2017 version of their reconstruction of Aristotle's *Protrepticus*, Hutchinson and Johnson applied these lessons to *DCMS* Chapters 22, 23, 26, and 27, distinguishing what they take to be quotation (or paraphrase) by Iamblichus from what they recognize to be Iamblichus' own scanty editorial contributions of introductory and concluding comments and bridge material.

The major attempt to interpret Iamblichus' particular intentions and to describe his overall Pythagorean project and the influences on it remains Dominic O'Meara's *Pythagoras Revived* of 1989.[31] But he often points out that it is hard to be sure of Iamblichus' particular conclusions, and I think this is partly because quotation so far outweighs editorial guidance. Quotation with editorial comment is one of the requirements we would make nowadays for supporting re-interpretation of earlier philosophy. Hutchinson has (in personal

communication) described these works of Iamblichus as text books for his students and Iamblichus as editor, not author. Because the editorial comment is sparse, and needs identifying, it is difficult to make out the particular lessons Iamblichus might have intended to draw for his students from particular quotations.

O'Meara makes Iamblichus' overall intention clear, however. He sought to present Plato and Aristotle as Pythagorean in spirit, and as following Pythagorean doctrines. Of the predecessors in this project described by O'Meara I will mention three. One was Numenius in the second century CE, who could have interested Iamblichus as coming from Apamea. He took it that Plato's successors had lost the Pythagorean truth which Plato secretively intended and that Plato's true Pythagoreanism must be revealed and restored. A second predecessor from the same period was Nicomachus of Gerasa, whose Pythagoreanizing *Introduction to Arithmetic* Iamblichus provides as the fourth in his Pythagorean sequence of books, a simple reiteration of Chapters 8 and following, together with an expansion of the material Nicomachus had covered in Chapters 1–7, by replacing his words with selections from earlier authors that make the apposite point more fully. Nicomachus is credited also with a Pythagoreanizing *Theology of Arithmetic* (*Theologoumena arithmeticae*), summarized by Photius in the ninth century, and perhaps excerpted in an anonymous tract of the same title. In this Nicomachus identifies Plato's ideal Forms with the categories of quantity, quality, etc. in Aristotle's *Categories* and makes both into characteristics of number, while the status given by Plato's *Timaeus* to number is reversed, when it is elevated above these Forms as what gives order to the material cosmos.

DCMS and the Pythagorean sequence as showing the value for Pythagoreans of mathematics for many reasons, including for life as a whole

The lost books of Iamblichus' Pythagorean sequence, judging from the excerpts preserved by the eleventh-century historian and philosopher Michael Psellus, discussed, besides arithmetic, the other branches of mathematics, geometry, music, and astronomy, and also discussed arithmetic in relation to physics,

ethics, and theology. Already in *DCMS* Iamblichus, drawing often on later Neopythagorean sources which, however, he takes to include Pythagoras and pre-Platonic Pythagoreans, speaks of the value of mathematics for life, for human and divine goods, and for the arts and crafts. Mathematics pervades philosophy, draws the mathematician upwards to intelligible beings and theology, shows the natural scientist analogies between the mathematical and natural worlds and provides orderly models for the statesman and ethicist (chs 15–16, repeated regarding intelligible beings, Chapter 24 and elsewhere). But the Pythagoreans also allowed for students who wanted only the mathematics, not the philosophy (Chapter 18). The extension to theology is through analogies of mathematical features with different gods. Attention is drawn also to Platonic intelligible Forms and the human soul is defined as a self-moving mathematical entity. In application to the mathematical features even of physical entities, mathematics raises the soul to more complete activity (Chapter 19). It purifies the eye of the soul and assimilates to the intelligible Forms and teaches their kinship with the Good. It gives the soul clarity, subtlety, accuracy, harmony freedom from passion, and converts it to the world of being, while providing advantages in the conduct of soul, the organization of cities, management of homes, the exercise of skilled work and in equipment for war and peace (Chapter 22). It provides a free-minded education without prior experience, offers exact proofs, supports crafts and natural science and makes us akin to the orderly and fine. But it is also valued for its own sake for its exactitude, for supplying first principles, for applying to the divine stars, and for being akin to philosophy (Chapter 23). Chapter 26 examines the case *against* the utility of mathematics and in reply again stresses the ease of acquiring it. Mathematics applies to every aspect of philosophy and life with the goal of the fine and the good and access to the realm of true being. But it also descends to the construction of political constitutions, to ethical questions and standards for households and cities, moral improvement and beneficial products of crafts (Chapter 30). It is illustrated in how many different ways mathematics can be applied to perceptibles below itself, but it can also be drawn upwards to things above itself (Chapter 32). The Pythagoreans saw mathematics as revealing the real beings above it, which would in turn explain ethical consensus and reveal the nature of good and evil and reveal the order in celestial motion and provide representations of the workings of nature (Chapter 34).

History retold by Iamblichus also in his commentaries

A third influence on Iamblichus' Pythagorizing was Pseudo-Archytas.[32] But this influence is reported as applying instead to Iamblichus' lost commentary on Aristotle's *Categories*. This commentary was evidently startling, judging from the records of it. Simplicius' commentary on Aristotle's *Categories* at 2,9–25, says that Iamblichus closely followed Porphyry's lost commentary on the work, but inserted two kinds of revision throughout. One addition was the Neopythagoreanism of Pseudo-Archytas' scheme of categories, which purported to be an account of categories by Aristotle's predecessor Archytas of Tarentum, and hence to be an early Pythagorean source used by Aristotle for his own scheme of categories. However, Iamblichus' acquiescence in this impersonation did not entirely sweep the field. The excellent Aristotelian commentator Themistius who flourished in Constantinople from the late 340s, about fifteen years after Iamblichus' death, until 384/5 CE, rejected the chronological claim of Pseudo-Archytas. Iamblichus' other addition in his commentary on Aristotle's *Categories* was his so-called 'intellective' theory of the categories, of which John Dillon's classic account has recently been reprinted.[33] The theory claimed that Aristotle's categories, including the category of substance, applied to Plato's superior realm of ideal Forms, as well as to every other level of reality. One example of Iamblichus' intellective theory of categories is provided by his analogues of Aristotle's categories of When and Where, the categories of Time and Place, which turns them into entities with dynamic force, including a Platonic Form of Time, which he uses to solve Pseudo-Archytas' version of the puzzles proffered by Aristotle in his *Physics*, 4.10, about whether time is real.

Iamblichus produced another major change in Platonism by connecting it with immensely more complex hierarchies of higher entities than before. The account of the created universe in Plato's *Timaeus* had recognized two higher levels, a divine Craftsman-Intellect shaping the world, and a World Soul making the stars rotate. Above this Soul and Intellect, the One was added by the thinker recognized by modern scholarship as founding Neoplatonism, Plotinus in his writings collected by Porphyry as *The Enneads*, although we shall see below Iamblichus offering suggestive hints from much earlier in his quotations from Plato's immediate successor Speusippus in Chapters 3 and 4

of *DCMS*. By surveying especially the writings of the later Athenian Neoplatonists, John Dillon has assembled fragments of Iamblichus' commentaries on Plato[34] which he describes as crowding these realms with extra entities. In some cases, he suggests, the number of higher entities may have lent support to Iamblichus' claims for the need for a priestly theurgist, or god-worker, to make the many divinities more accessible to humans.[35]

The influence of *DCMS* on the fifth-century Athenian Neoplatonists Syrianus and Proclus

Dominic O'Meara's *Pythagoras Revived* has described the influence of *DCMS* on two successive heads of the fifth-century CE Athenian Neoplatonist school, Syrianus and Proclus.[36] Syrianus (head from about 432 to 437) wrote a commentary on Aristotle's *Metaphysics*, Books 3–4 and 13–14, translated in two volumes in the present series by John Dillon and Dominic O'Meara, in which he sought to defend from Aristotle's criticisms Plato's theory of intelligible Forms, a theory that he saw as coherent with Pythagoreanism (81,31–83,31) as well as Platonic. For fuller exposition he recommended the collections of Pythagorean doctrines by Nicomachus and the treatises of Iamblichus, whom he saw as 'divine' (103,4–10), unlike Aristotle (168,2–14), although on other subjects he regarded Aristotle as a 'benefactor of mankind' (80,10–16). He also provided a digest of the chapter headings of Iamblichus' *DCMS*. He regards the human soul as containing a median level of Forms and as being structured by geometrical, arithmetical, and harmonic proportions (82,20–83,7), so that by being reminded of its median Forms, the soul is able to recognize perceptible objects. Objects of mathematics are intermediate between perceptible and intelligible reality (4,5–11).

Syrianus' successor Proclus (412–485 CE) chose Euclid instead of Nicomachus as his model in mathematics and wrote a commentary which in its surviving form covers Book 1 of Euclid's *Elements*, and whose first prologue corresponds to Iamblichus' *DCMS*. In Chapter 6, Proclus develops the objections of Iamblichus and Syrianus against Aristotle's view of mathematical entities as our abstractions from physical entities and treats them instead as projections of innate intelligible principles. But he substitutes Plato for

Iamblichus' Pythagoreans 'Archytas' or Brotinus, as the true source of Iamblichus' ideas and as influencing Euclid. Like Iamblichus in his Pythagorean sequence, he uses mathematics as a key to physics and ethics and as an initiation to theology and metaphysics.

Identifications so far of the sources of *DCMS*

I have noticed a nice sharp set of distinctions at the beginning of *DCMS* Chapter 32 among the ways in which mathematics can be applied to perceptible objects, starting with Aristotle's own distinctive example of abstraction, which for once is not subjected to demurral. Given the sharpness of the distinctions and the unusual acquiescence in Aristotle's idea of abstraction, I should not be surprised if this short passage turned out, like a good number of others, to derive from Aristotle, contrasting, as it does, with the rather sketchy comments that follow in much of this chapter and the next. I am sure that with the progress made by Hutchinson and Johnson, there will be further advances in the identification of sources.

Many references to Plato, his followers, and to Pythagoreans or Neopythagoreans have been identified by others, or by Iamblichus himself. But I will finish by indicating the main published suggestions at the time of writing (more fully reported in my notes to the translation) about embedded quotations in *DCMS* particularly from Aristotle. One of the most novel concerns Chapter 27 and its overlap with Aristotle's *On Parts of Animals* 1.1. Hutchinson and Johnson argue in one of their two 2017 online publications that Aristotle's *On Parts of Animals* and this section of Iamblichus' *DCMS* are alike adaptations of a lost section of Aristotle's *Exhortation to Philosophy*. For one thing, it would be an unlikely project for Iamblichus to explain the methodology of the exact science of mathematics by *starting* from *On Parts of Animals* with its methodology of the least mathematical of the sciences, zoology. By contrast, the recognized fragments of Aristotle's *Exhortation to Philosophy* could easily be converted to a source for the support of different subjects: both zoology in Aristotle's *On Parts of Animals*, and later theoretical mathematics in Iamblichus' *DCMS*. A second consideration, besides the unlikelihood of Iamblichus arguing from zoology to mathematics, is the presence of an exhortation to study zoology

in Aristotle's *On Parts of Animals* at 1.5. Aristotle had in his *Exhortation to Philosophy* argued that beauty exists in the mathematical sciences. Here in *On Parts of Animals* 1.5, he takes it for granted that the objects at least of astronomy are divine and worthy, but he says he has already discussed the philosophy of divine things. So now it remains to speak of animals, leaving out none, whether more or less worthy. Aristotle opposes the claims of those who are childishly disgusted at the study of the less worthy animals, since in all natural things there is something wonderful, the purpose of their parts. The philosopher Heraclitus said, 'There are gods even in the kitchen'. Iamblichus similarly argues in the second book of his Pythagorean sequence, his own *Exhortation to Philosophy* ch. 9, 50,16–51,4, that anything that comes into being, or has come into being by nature, comes into being, or has come into being beautifully. For what comes into being in accordance with nature, comes into being for a purpose. Thus for a second time, we see some of the same ideas being canvassed in Aristotle's *Parts of Animals* and in a work of Iamblichus that has for some time been seen as in parts quoting Aristotle's *Exhortation to Philosophy*.

The point in *DCMS* Chapter 27 that one should accept plausible statements from the orator but require necessary statements from the dialectician was pointed out by Hutchinson and Johnson to be a version of what Aristotle says in the passage at *Nicomachean Ethics* 1.3, 1094b25–7, in his explanation of why statements in ethics hold true only for the most part. The *DCMS* even uses similar language in saying that the type of proof should match the underlying matter, i.e. subject matter (*DCMS* says 'underlying substance'). The view of Hutchinson and Johnson is that Aristotle draws on his earlier *Exhortation* in later works including both *Nicomachean Ethics* and *On Parts of Animals*.

Hutchinson and Johnson in 2017 drew attention to at least three more cases of quotations drawing on Aristotle in *DCMS* between Chapters 21 and 27. Much of *DCMS* Chapter 26 was already was included by Rose in 1886 in his collection of fragments of Aristotle's *Exhortation to Philosophy*. The *DCMS* subject is objections to theoretical mathematics and replies. Hutchinson and Johnson studied the textual overlap in Iamblichus' citations from Aristotle in *DCMS* Chapter 26 and *Protrepticus* Chapter 6 and found that the *DCMS* version is a condensed reiteration of the *Protrepticus* version (see details in the note to the translation of that passage). The identification of Isocrates as a speaker in Aristotle's work together with the reconstruction of the dialectic

between objections to theoretical philosophy and protreptic replies, is the topic of Hutchinson and Johnson's most recent publication, 'Protreptic and Apotreptic: Aristotle's dialogue *Protrepticus*'.[37]

They argue that both of these two works of Iamblichus could in turn be quoting from the same earlier source identified by Jaeger and Merlan – Aristotle's *Exhortation* – and that Aristotle had there put objections to theoretical mathematics into the mouth of Isocrates and had put replies into his own mouth.

In *DCMS* Chapter 23, Merlan knew that two passages which Iamblichus quotes in Chapter 26 had already been included by Rose in 1886 in the collection of fragments he identified as originating from Aristotle's *Exhortation*. He therefore argued that the replies in Chapter 23 could reasonably be taken to be drawn from Aristotle's *Exhortation* as well. He added that *DCMS* Chapter 23 is similar once again to Chapters 6 and 9 of Iamblichus' own *Exhortation to Philosophy*. And these last Iamblichan chapters in turn contain two excerpts from Aristotle's own *Exhortation*, fragments 5 and 13 in Walzer's reconstruction.[38]

As regards Chapter 22 of *DCMS*, Hutchinson and Johnson in their 2017 reconstruction online of Aristotle's *Exhortation*, take much of 67,7–68,4 to be quotations, interrupted by Iamblichus' comments, from a speech put into the mouth of Heraclides in Aristotle's *Exhortation to Philosophy*.

A *different* overlap of *DCMS* Chapter 25 noticed by W. Burkert has been pointed out to me by Hutchinson, as noted in that chapter, with Iamblichus' *first* work in his Pythagorean series, *On the Pythagorean Life*, Chapter 18, and Burkert says that some of Iamblichus *On the Pythagorean Life* is drawn in turn from Aristotle's own account of the two types of Pythagorean in Aristotle's fragmentary *On the Pythagoreans*.

Aristotle is also the source, according to T. Orlandi, of the material in Chapters 16 and 17, where Iamblichus relies on an early work of Aristotle, he suggests, either directly or via an intermediate epitome.[39]

Finally, I have noticed a nice sharp set of distinctions at the beginning of *DCMS* Chapter 32 among the ways in which mathematics can be applied to perceptible objects, starting with Aristotle's own distinctive example of abstraction, which for once is not subjected to demurral. I should not be surprised if this short passage also turned out to derive from Aristotle, contrasting, as it

does, with the rather sketchy comments that follow in much of this chapter and the next. I am sure that with the progress made by Hutchinson and Johnson, there will be further advances in the identification of sources.

Notes

1 John Dillon, 'Iamblichus of Chalcis, (240–325 A.D.)', *Aufstieg und Niedergang der Römischen Welt* II.36.2, pp. 862–909. (He has a shorter account in 'Iamblichus of Chalcis and his school', in Lloyd P. Gerson, ed., *The Cambridge History of Philosophy in Late Antiquity*, 2 vols (Cambridge: Cambridge University Press, 2010), vol. 1, ch. 19, pp. 358–74.)

2 Eunapius, *Lives of the Sophists*, ed. W. C. Wright, Loeb edition, pp. 458–9.

3 Translated in the present series, Ancient Commentators on Aristotle, by Gillian Clark. See also Richard Sorabji, *Animal Minds and Human Morals* (London: Duckworth, 1993), pp. 182–7.

4 Porphyry, *On Abstinence from Killing Animals* 2.34, 35, 42–3, 61.

5 Richard Sorabji, *Aristotle Transformed* (London: Duckworth, 1990), pp. 10–11; second edition (London: Bloomsbury, 2016), pp. 11–12; *Animal Minds and Human Morals*, pp. 185–7.

6 Richard Sorabji, *Aristotle Transformed* (1990), pp. 9–10, with illustration; second edition (2016), pp. 9–11.

7 Discussed in Richard Sorabji, *Emotion and Peace of Mind* (Oxford and Chicago University Presses 2000), pp. 284–7, and *Animal Minds and Human Morals*, pp. 187–8. Iamblichus' *Mysteries* is translated by Emma C. Clarke, John M. Dillon, and Jackson P. Hershbell (Atlanta: Society of Biblical Literature, 2003).

8 Iamblichus, *Mysteries* 7.4, 255,5–263,3, 2.11, 96,13, 97,9, and according to Proclus *On Plato's Timaeus*, 1, 209,1–212,6, and *On Plato's Cratylus*, 31,24–32,5 = texts 18b13, 18b2, 18c5, and 18b14 in Richard Sorabji, *The Philosophy of the Commentators, A Sourcebook*, vol.1 (Duckworth, London and Cornell University Press, 2005).

9 Iamblichus, *Mysteries* 1.11, 39,14–40,8.

10 So *Anonymous Prolegomena to the Philosophy of Plato*, ch. 26, lines 16–24, ed. Westerink. John Dillon has translated the fragments on Plato's Dialogues (Leiden: Brill, 1973).

11 Translated by John Dillon and Dominic O'Meara in this series.

12 See e.g. *Ten Problems Concerning Providence*, translated in this series.

13 Translated in this series.

14 Already partly translated in the present series, with the rest in preparation.

15 Damascius, the last Athenian head in Athens up to 529 CE as well as the Athenian author of the commentary on Aristotle *On the Soul*, ascribed to one of Damascius' junior colleagues, either Simplicius or, as more widely thought, Priscian, and translated in this series.

16 Ammonius, *in Int.* 135,14, relating to Ammonius' discussion at 132,8–137,11, translated in this series.

17 By Riccardo Chiaradonna and Marwan Rashed.

18 Gabor Betegh, 'The transmission of ancient wisdom', in Lloyd P. Gerson, ed., *The Cambridge History of Philosophy in Late* Antiquity, 2 vols (Cambridge: Cambridge University Press, 2010), vol. 1, p. 31; Dominic O'Meara, *Pythagoras Revived* (Oxford: Oxford University Press, 1989), p. 110.

19 Damascius, *On Plato's Phaedo*, 1. 172, ed. Westerink.

20 A version of the lectures by Hermeias' son, Ammonius, on Aristotle *On the Soul*, written up with his own comments by Philoponus, makes only one reference, and Olympiodorus later in the sixth century, makes only two references in his commentary on Aristotle's *Categories*.

21 Richard Sorabji, *Aristotle Transformed*, p. 12. Ammonius' pupils and editors Asclepius and Philoponus felt free to include in the syllabus also a commentary on Nicomachus of Gerasa's *Introduction to Arithmetic*, of which Iamblichus had supplied edited quotations as the fourth book of his Pythagorean sequence. That of Asclepius is edited with introduction and notes by Leonardo Tarán, *Transactions of the American Philosophical Society*, n.s. 59, Philadelphia 1969, in eighty-nine pages; that of Philoponus is edited by R. Hoche, 1864–7; Italian translation by Giovanna Giardina, *Tra neopitagorismo e neoplatonismo*: *commentario alla introduzione aritmetica di Nicomaco di Gerasa* (Catania: CUECM, 1999).

22 Anne Sheppard, 'Proclus' Philosophical Exegesis: The Use of Aristotle and the Stoics in the Commentary on the *Cratylus*', in J. Pépin and H.-D Saffrey, eds, *Proclus*, *Lecteur et interprète des anciens* (Paris: CNRS, 1987), pp. 137–51.

23 Richard Sorabji, 'Divine Names and Sordid Deals in Ammonius' Alexandria', in A. Smith, ed., *The Philosopher and Society in Late Antiquity*: *Essays in Honour of Peter Brown* (Swansea: The Classical Press of Wales, 2005), pp. 203–13, developing Richard Sorabji, *The Philosophy of the Commentators*, *200–600* AD, *A Sourcebook*, 3 vols (London: Duckworth, 2004), pp. 21–5 of the introduction in each of the three volumes.

24 Translated by J. M. Dillon and J. Hershbell, *Iamblichus: On the Pythagorean Way of Life* (Atlanta: Society of Biblical Literature, 1991) and by Gillian Clark, *On the Pythagorean Life* (Liverpool: Liverpool University Press, 1989).

25 Translation commissioned for the present series from D. S. Hutchinson and M. R. Johnson.
26 Dominic O'Meara, *Pythagoras Revived*: *Mathematics and Philosophy in Late Antiquity* (Oxford: Oxford University Press, 1989).
27 See Doug Hutchinson and Monte Johnson, 'Authenticating Aristotle's *Protrepticus*', *Oxford Studies in Ancient Philosophy* 29 (2005), 193–294.
28 See Ingram Bywater, 'On a Lost Dialogue of Aristotle', *The Journal of Philology* 2 (1869), 55–69, and Valentinus Rose, *Aristotelis qui ferebantur librorum Fragmenta* (Leipzig: Teubner, 1886).
29 See John Dillon, 'Speusippus in Iamblichus', *Phronesis* 29 (1984), 325–32, at 329.
30 'Iamblichus of Chalcis (*c*. 240–325 A.D.)', *Aufstieg und Niedergang der römischen Welt* II.36.2 (1987), pp. 862–909, at 876.
31 Dominic O'Meara, *Pythagoras Revived*: *Mathematics and Philosophy in Late Antiquity* (Oxford: Oxford University Press, 1989).
32 That Pseudo-Archytas was an influence on Iamblichus is clear from Iamblichus own mention of Archytas and citation of the work *On Wisdom* in Chapter 4 of the *Protrepticus*. It is significant that this is virtually the only source for which Iamblichus names both the author and title of the work.
33 John Dillon, 'Iamblichus' *noera theôria* of Aristotle's *Categories*', *Syllecta Classica* 8 (1997), 65–77; repr. in Richard Sorabji, ed., *Aristotle Re-interpreted* (London: Bloomsbury, 2016), pp. 313–25.
34 John Dillon, *Iamblichi Chalcidensis in Platonis Dialogos Commentariorum Fragmenta*, (Leiden: Brill, 1973).
35 John Dillon, as above, 'Iamblichus of Chalcis and his school' in Lloyd Gerson, ed., *The Cambridge History of Philosophy in Late Antiquity*, vol.1, pp. 358–74.
36 Syrianus in ch. 6, esp pp. 120–33; Proclus in ch. 8, esp. pp. 157–74.
37 D. Hutchinson and M. Johnson, 'Protreptic and Apotreptic: Aristotle's dialogue *Protrepticus*', in O. Alieva, A. Kotze and S. Van der Meeren, eds, *When Wisdom Calls: Philosophical Protreptic in Antiquity* (Turnhout: Brepols, 2018), pp. 111–54.
38 See Richard Walzer, *Aristotelis dialogorum fragmenta* (Firenze: Sansoni, 1934).
39 T. Orlandi, 'Il primo Aristotele nel *De communi mathematica scientia* di Jamblico', *Rivista Critica di Storia della Filosofia* 18 (1963), 592–98.

Textual Emendations

The following list notes departures from the Greek text in N. Festa's 1891 Teubner edition, as reprinted in 1975 with additional material by U. Klein. The main text is identical in both editions.

13,26: accepting the Add. Steel, 'some' to secure an acceptable sense.

28,14: reading, with ms. P, 'visible' (*horatōi*) instead of Festa's *ahoratōi*.

29,1: reading *pasi*, *pasa* in the text, corrected in the apparatus.

34,12: adding of 'both plurality', accepting a conjecture of Festa.

35,3: adopting the proposal of Festa to insert *apodeiktikon* after the otherwise meaningless *ton*.

36,12–13: reading in effect, with Vitelli, *poiei ta theôroumena* for the untranslatable *tauta sumphona poiêta theôroumena di'autôn alathea*.

36,16: reading *kathaper gar gramman dikha tetmamenên anisa tmêmata kai palin hekaterôn tmêmata tetmêmena ana ton auton logon* in place of the text's *kathaper gar gramman dikha tetmamenên kai isa palin hekaterôn tmêmata tetmamena ana ton auton logon*.

37,11: reading, with Festa, <*ouk arkhas*> *all' hormas te kai epibasias* instead of the untranslatable *hypothesias all' hypothesias all' arkhas te kai epibasias*.

42,14: reading *apogennômenê* instead of *apogennômenês.*

67,26: adopting Festa's suggestion of omitting the *to* before *on*.

70,19: suggesting some such phrase as *touto de tais* for filling a small lacuna.

94,26: reading *touto to* instead of *tou to*.

Iamblichus

On the General Science of Mathematics

Translation

Iamblichus of Chalcis in Coele Syria on the General Science of Mathematics

Chapters of the Third Discourse[1]

1. What is the aim as a whole of the present book and what are the particular targets under that aim, divided into what topics, and whence do they derive the primary grounds of their special study, and from what sort of essential being?[2]

2. What is the whole theory behind mathematics in general, and what the nature of mathematical science, whence it derives, and from what sources its definition is to be acquired; also how great is its compass, and into how many general kinds it extends.

3. What the principles of the whole of mathematics are and how they differ from other principles which are of other subject-matters; also how such sorts of principle provide the ground common to the whole of mathematics.

4. What the special principles of each branch of mathematics are and what their special character, their differences from each other and from all the other principles of all existent things.

5. What common features underlie all the mathematical sciences with which students of mathematics are engaged, and how it is possible to study them universally.

6. What is the best use of the study of mathematics, and to what goal one should refer the best sort of study of it.

7. What the special object of knowledge is which is the basis of each branch of mathematics, and how it is possible by division to make a general distinction of them, so as to know both the one and the manifold in mathematics, its character, and how it should be defined.

8. What is the general criterion for the whole of mathematics and how it is discovered from the division of the line which the Pythagoreans hand down.

9. About those that propose a definition of the essence of mathematics, of whom the first viewpoint is presented as being that of those who relate it to the

soul; several causes of such a conception are stated, and starting points for the whole study of mathematics.

10. How the essence *(ousia*) of the soul is composed out of all the mathematical sciences, and according to what distinction the mixture of them within it may be determined, and whether the soul encompasses within itself the whole level of being of mathematics, or if some other principle of mathematics is also to be considered.

11. What is the function (*ergon*) of the study of mathematics, and how it arises, and that it is called mathematics conformably to these.

12. What are the capacities (*dynameis*) of mathematical science, the orders within them, the number of differences by which they are divided, and in how many ways they are conceived.

13. What elements (*stoikheia*) and kinds (*gene*) of mathematical science there are, and how the same ones are present as elements and how as kinds; by what these are different from those in the other sciences and kinds of things, both the intelligible and such as are in the realm of generation.

14. Concerning the similarities and dissimilarities in mathematics, what they are and how far they extend; how they are present in the subject-matter of mathematics, and in what respect they differ from similarly named kinds, both of intelligible and of sensible objects.

15. How the whole of mathematical science extends, both itself, its kinds, its elements, and its principles, into the whole of philosophy and all the parts of philosophy; how it is involved with them and by what sort of contribution.

16. How many goods it provides to the arts and crafts, universally to them as a whole and to their different kinds, as it does to the theoretical, creative, and practical crafts; and succinct instruction about them.

17. What the order of presentation in mathematics is, and if it has a natural order also for learning; and if each ordering is appropriate to each, and the two to each other.

18. The special methods of the Pythagorean presentation of the mathematical sciences, how and in what matters they made use of them, and for whom; also that they always took due account both of the subject-matter and of learners.

19. The Pythagorean division of mathematical science as a whole into genera and the most important species, which creates a common study of them.

20. What is the method of definition in mathematics and how it comes about; the kind of help it contributes to science; that mathematics has a goal and what sort of goal it has.

21. Who were the originators of Pythagorean mathematics, and what according to him are the salient points of such a science; how, according to him, one should organize mathematics. A general survey.

22. The distinctive practice of mathematical science according to Pythagoras, and how many uses of it he envisaged for the soul and for mankind; also how they practised it throughout the whole of their own life.

23. That the Pythagoreans extended mathematics not blindly, but with a view to the life of necessary utility; a record of the reasons for this, with many examples.

24. What was the group practice in the Pythagoreans' occupation with mathematical pursuits and what was their training and preparation in sciences?

25. Which of the Pythagoreans were Mathematicals, and in what respect did they differ from the Acousmatics? What was their function and of what kind the form of their arguments and demonstrations?

26. Objections to mathematics as being worthless; replies thereto, and comparisons of these arguments in detail.

27. What the really educated person should ask for from the mathematician; also how one should judge his study and by what standards determine its correctness.

28. When the problem or the method of demonstration is mathematical or belongs to another science; a scientific distinction.

29. Concerning the mathematical syllogisms, and the mathematical divisions and definitions; how mathematical science should use them, whether it gets its principles in its own way or from dialectic.

30. That mathematics makes great contributions to philosophy in general and to all its branches, supporting it in every respect; and especially that of the Pythagoreans, which is much superior to other mathematics.

31. That the Pythagoreans used the same mathematics for many different purposes; that they conducted many studies that clarified the same problem, and for what reasons they did that.

32. How sometimes we tackle also questions about perceptible things mathematically, and how often this occurs; also how in mathematics many things lead on to others and why this is so.

33. What is common to the whole of mathematical science and specific to it with regard to the differences observed in its many species, and how one should divide it in accordance with scientific division from one into two and then more species.

34. Whence mathematical science got its name and what its characteristic is; also to what one should pay attention in judging the species of mathematics.

35. Summary of the general account of all mathematics, a justification of the order of the chapters, and also a reminder at the same time about the right way to divide up the whole conspectus of the discipline.

Chapter 1[3]

What is the aim as a whole of the present book and what are the particular targets[4] under that aim, divided into what topics, and whence do they derive the primary grounds of their special study, and from what sort of essential being?[5]

9,4 The object of the present investigation is to indicate the general study of mathematics, what it is as a whole, and what single ground and fundamental subject-matter it has.[6] Following on this first topic, let us investigate whether

there are perhaps two principles of it,[7] and, after this dichotomy, we shall attempt to work out whether there is some determinate number of kinds of 10
mathematics by means of a scientific division.[8] Then we shall also investigate the common species of all mathematics from some common point of focus,[9] though without yet dealing with individual theorems.[10] In each of the above we shall indicate[11] the subject-matter with which each kind and species of mathematics is concerned and what each of them contributes to the totality, 15
and we shall not omit a discussion of their relation to each other, what their kinship is and whence it comes, by what principles they are bound together, and upon what more basic principles of the study they depend; also how one 20
may discover them; also, of what use the subject is[12] and to how many good things it leads; also, that it is both choiceworthy in itself and also through the sciences arising from it; also, that it converts[13] the reasoning faculty[14] to the 10,1
whole of philosophy and to the whole science of things with true being that are intelligible. That is the sum of the things that lie before us to deal with in this book. Let us start by taking up the first of the topics mentioned above. 5

Let the following postulates be assumed in common for the totality of the objects of mathematics: that their mode of existence is incorporeal and self-subsistent, holding an intermediate rank between indivisible beings and those 10
that are divided about bodies,[15] both as forms and as reason-principles,[16] being assigned the middle status between the simple and the divisible, being purer than the latter, more variegated than the former; that they make use of composition[17] and division, but oversee the synthesized and the divided without coming to be and eternally; they are inferior to intelligible beings but 15
prior to natural ones; they are in beauty, in order, and in exactitude superior to things visible but inferior to the intelligible, and similarly they have an intermediate symmetry and compatibility; they have the power to transport[18] and lead over to the indivisible forms, since they are akin to them, and they 20
lead away from corporeal concerns those who have become accustomed to them, converting them to the divine beings, as if on a ladder leading up to the heights.

But one must view the gift of incorporeal essence not as coming to them in 11,1
a secondary way from only a single kind of being, but from all kinds whatsoever that inhere in true being and Intellect. For from all these there descends to the intermediate natures of the mathematical objects the mediating role between

the causal principles and their products, and connects things in the realm of generation to things that have true being,[19] and brings about their community with each other.

So then, the study of mathematical objects being so wide and thus extending in this way to all things, mathematical science is an intermediate kind of knowledge, broader than intellection through making compositions, being a discursive kind of reasoning encompassing many things in one, using rather detailed descriptions and explanations, and forms and reason-principles that are intermediate and not altogether limited, but which mark off the limit around the unlimited, and which provide clarity in matters that are not altogether knowable.

Chapter 2

What[20] *is the whole theory*[21] *behind mathematics in general, and what is the nature of mathematical science, whence it derives, and from what sources its definition*[22] *is to be acquired; also how great is its compass, and into how many general kinds it extends.*

Since the science is of this character, those should undertake it who are prepared to turn away from bodies and coming to be, who have purified themselves from imaginations and sensations, accustomed themselves to the essentially incorporeal, and are given to the continuous practice of rational arguments. It is reasonable to ascribe to them a definition[23] in terms of the science of being, pure intellection, pure reasonings, forms without matter and the delimited truth of things intelligible.[24] For it is from these that one may grasp the perfection and clarity of the knowledge that is within them. This knowledge of theirs extends to all things that are intermediate genera and species of being, such as are contained within definite numbers and such as proceed in a definite way in accordance with certain specific differentiae. And
12,1 of these some progress forward and rise to the heights, others approach the inferior and lower kinds of thing, while those which remain between these join the extremes together. Its genera and species must be distinguished in all those respects, and also some are to be distinguished as self-subsistent, others as

related to other entities. One must also be aware of the distinction between them in respect to quantity, and also in respect to the divisions of their intermediate reason-principles and forms.[25] Also some of them are to be taken as prior, others as posterior, in order that their natures may be mutually ordered. One can also argue to their difference with regard to such cognitive powers of the soul as are intermediate and discursive, as Archytas[26] appears to do in his treatise on the division of the cognitive line.

Let such, then, be in outline the first inclusive account of the general theory of mathematics; and following on that, let us start again from the beginning and attempt to deal with the already mentioned problems one at a time.

Chapter 3

What the principles of the whole of mathematics are and how they differ from other principles which are of other subject-matters; also how such sorts of principle provide the ground common to the whole of mathematics.

Let us distinguish with regard to the whole of mathematics in general what the principles of the subject-matter of mathematical science are. For since every science arises from its own principles, so also understanding of the subject-matter of mathematics would do best to take its start from this.[27]

So, as is the view of the Pythagoreans, it is clear to everyone that the limited and the unlimited are the principles of all the mathematical sciences, and of the whole essence of mathematics. But, they say, the account of each of these is not single nor the same for all kinds of things; but, in the case of the intelligible 13,1
forms and immaterial reason-principles, such principles are of one sort, altogether intelligible, immaterial, and indivisible in their own nature, but in the case of mathematical plurality and magnitude, they will be the causes of division and separation; also they will partake of a divisible nature, and take on the genera that belong properly to the whole of mathematics; they will also share in composition and be distinguished by a different sort of reasoning[28] from that power which distinguishes simple, indivisible, and intelligible things. Perhaps some – those who suppose these principles to be in the soul and within the soul's vitality and powers – will confer change on these principles of

mathematics.[29] But it is better to place the soul in another category of being[30] and to suppose the principles of mathematics and its subject-matter to be unchangeable; for their forms are always stable and we contemplate them as constant and the same.

So these principles of the unlimited and the limit[31] are of an intermediate nature, with the form of the limit always prevailing over the unlimited and binding it within itself; wherefore it both always proceeds towards the unlimited, but yet is endowed with definition by the limiting principle. Therefore these principles differ from those present in the intellect by providing from themselves a cause for division, plurality, magnitude, and composition; but they are distinguished from the reason-principles in nature and in the soul both by being unchanging and because of the incorporeal intermediate entities, <some>[32] [*sc*. mathematical entities] subsist on their own, separated from matter, while others [*sc*. soul and nature] are attached to matter.

14,1 So one might be persuaded by these considerations that they are distinct from other principles. On the other hand, one must recognize their common nature which extends to all things from their intermediacy thus simply conceived and from their nature, lower than that of the simple and intelligible forms, but superior to those ones that are divided up between bodies. Also, if one were to focus on reason-principles, the consideration of reason-principles must also be viewed from the perspective of this common nature of theirs. Also one must suppose that indeterminacy extends generally to them all; and if some receptacles of mathematical forms are discerned by intellection, one may properly leave these as common to the many-faceted structure which mathematics may be seen to be. For in that way one would come to recognize what is common to mathematics, which is hard to grasp so as to include it in one calculation, because it is present in many disparate things and thus scarcely comprehensible.

So much, then,[33] for our distinction of common features.

Chapter 4

What the special principles of each branch of mathematics are and what their special character, their differences from each other and from all the other principles of all existent things.

But if we are also to distinguish the specific principles of each branch of mathematics, what they are, what they are like, and what their special character is, and how they differ from each other and from all the other principles of all things, now is the right time to discuss these matters.

Now the best approach of all, since there is an order among them and some take the lead as prior not only in order but also in nature, since they destroy but are not destroyed, introduce but are not introduced,[34] but others are inferior to these both in seniority and in simplicity, it is best of all for these reasons and fitting for us to follow their natural order as well, and to speak first about the first, and then similarly about the rest.[35]

So then,[36] one must posit two primary and supreme principles of numbers and all mathematical objects. There is the One, which one cannot even describe as *being*,[37] since it is simple and since it is the principle of all things, while the principle is not yet such as those things of which it is the principle; and then again there is another principle, that of plurality, which is as such able also to produce division, and which for that reason we might compare to some altogether moist and pliable matter, indicating thus an illustration as fitting as we can. From these, the One and the principle of plurality, is brought about the primary genus, since numbers are composed from both of these through the agency of some persuasive necessity.[38] Also, one should go through numbers one by one and declare that this nature imparts to every number both every division and magnitude, to speak universally, while that each of them has a definite quality and is also determinate and one, is impressed and stamped upon it by the undifferentiated and indivisible principle.

It is not, perhaps, appropriate to count a thing as bad or ugly[39] because it happens to be as such responsible for magnitude and division, and also of growth; for we are not accustomed to assign such a kind to an evil role in other connections, but rather sometimes we would perhaps truly call largeness, combined with a certain quality, a cause of magnificence and liberality, so that it would be far from fitting to call it bad. For if indeed one should happen to praise the nature of the One because of its self-sufficiency and responsibility for certain beautiful elements in numbers, how would it not be irrational to say that the bad or ugly was naturally receptive of such a thing?[40] For then it would come about that the bad and the ugly was not altogether blameable, since what is receptive of the praiseworthy must also be called praiseworthy. So that is

how we should conceive this principle <of plurality>. But as for the One, it is not fitting to call it either beautiful or good, since it is superior to both beauty and goodness. For, as nature proceeds further from the primary, first the beauty appeared, and second – when the elements are at a yet greater remove[41] – the good.

So then, the primal receptacle and magnitude, or whatever one should call it, printed out the class of numbers, indeterminate in plurality, as one would expect, but in a way determined in form as a legacy from the One.[42] Now, if one supposes some single unlimited material and receptacle for all things, there will come about, as it seems, the illogical result that, when the form of the One enters into it, since it is similar throughout, the kinds of product in their turn should not turn out the same. The result will be that all kinds of things will turn out to be numbers; for we shall not have a suitable differentia to apply to explain why the nature of numbers was generated at this point, and after that the nature of lines and planes and figures,[43] and not always the same kind of thing, since they came from similar elements interwoven with each other in the same way. But if one posits a single first cause of all plurality and magnitude, but one providing many differentiae within itself, on account of which it was of a nature to give birth to more and more different kinds throughout all nature, then, although the same nature of the One is present throughout, even this will not display its nature exactly throughout by reason of the grossness of the material principle, like a figure portrayed in random pieces of wood,[44] and so this might not unreasonably happen to the One. Now one might perhaps reasonably be unhappy that the first element should be divisible into so many different kinds, especially if it extended altogether throughout these exemplars; for everywhere, they say, the element is the most simple. So the only alternative left to us is to suppose some other cause of magnitude, and, as we posit the monad in numbers representing the One, so to posit the point for lines, while position and extension of places first emerge around the lines and areas and solids, and in the same way place also emerges through the difference of the receptacles, bestowing something special on the kind produced by it.[45] Also one would perhaps not be mistaken if one accounted for continuity and the baser and grosser element in numbers[46] by this difference. With these the second kind would be completed; for I place in the same kind lines and solids and the extension of areas. So the first matter is that of numbers, the second

that of lines, surfaces and solid figures. One must also presuppose their own receptacles for the other branches of mathematics in the same way, however many reason may discover.

Let that be our position on this matter. But[47] the elements of which 18,1
numbers[48] are composed are not as yet either beautiful or good. For number exists through the compounding of the one and the material cause of plurality, and in these first appear existence and beauty. Next, from the elements of lines 5
there appeared the geometrical level of being, in which in the same way there is being and beauty, among which there is nothing either ugly or evil. Finally, only at the lowest level, among the fourth and fifth levels of being,[49] which are 10
composed from the lowest level of elements, does evil arise, and even then only as an incidental consequence of their falling away and not controlling certain sides of their nature.

From these considerations[50] it is also clear what the difference is between mathematical principles and others. For they are superior to the lowest 15
ones because they are incorporeal while those are corporeal; they differ from those viewed as concerned with life, because they are unchanging while the others are characterized by change;[51] they differ from the intelligible because, while those pre-exist as simple, these exhibit a principle of composition and 20
division.

So[52] let that be our analysis of the general account of the principles of mathematical sciences and the special account of each of them. Let the ways in which they differ from other principles be distinguished in this manner.

Chapter 5

What common features underlie all the mathematical sciences with which students of mathematics[53] are engaged,[54] and how it is possible to study them universally.

Thus the subject-matters of the study of mathematics which extend in common 25
throughout the whole of this science are those theorems such as are common to all, capable of being adapted to numbers, to magnitudes and also to 19,1
harmonics, astronomy, and all its other parts. Such are those about ratios, those

concerning combinations and divisions in general, those studying[55] equality and inequality of whatever quantity or quality, those governing multiplication or separation, or excess and deficiency, or the determinate and indeterminate from a common point of focus,[56] or the self-substantiated and the relational, or quantity without qualification and without adding any particular kind of quantity, <and then> quantity <in relation to something else>, or[57] order and beauty in the species of mathematics from the scientific aspect, without further stipulating a particular type of beauty; for that sort of thing already relates to some subordinate science.[58]

Further, as for what looks to the firmness and constancy of mathematical science, which does not change from time to time, nor depart from its own essential nature, nor is conceived first in one way and then in another, this too is embraced by the common subject-matter of mathematical science within its reasoning. But one should not think of these common features as supervening on, but as being prior to, the particular studies; nor should they be thought of as included in the particular and having their being with them, but as having already a being which is prior to them and more fundamental, and not as dwelling in them but as set in place before the special objects of each mathematical science. For this reason knowledge of them is common to all and fundamental, more basic than that of the particular ones, and giving a general overview of all, arranging together all theorems of mathematics from a single beginning to a single end, surveying their kinship and likeness to each other, while taking account also of what is dissimilar and different in them, collecting together their primary kinds and their species, and distinguishing them. Also it surveys their common agreements, their prime hypotheses, their definitions, their postulates, their divisions and compositions, their aggregations and separations, their excesses and deficiencies and comparisons, throughout all the kinds of mathematical entities. It gives a unified account and not one separately of each, distinguishing what is possible in them and what impossible, what necessary and what not, the true and the false, and also their differences, their number and character, conducting a most accurate investigation.

These, then,[59] are the distinctions to be made for the present about the basic subject-matter of mathematical science and the general approach to its study.

Chapter 6[60]

What is the best use of the study of mathematics, and to what goal one should refer the best sort of study of it.

It is to be understood about all such studies that if one undertakes each of them correctly there is a great advantage to him who approaches them in the right way; otherwise it is better to call on the deity. The right approach is as 25
follows – for we must specify at least this much. Every diagram, numerical 21,1
system, and the whole construction of harmony, being the analogue of the orbiting of the stars, should be manifest as being a single system of them all to him who studies them in the right way. And it will be so revealed to him who 5
learns rightly to regard what we say. For, to those who think things through, a single bond will be revealed between the natures of them all. If anyone undertakes this in some other way he must call upon fortune, as we also say; for without these there would never be a happy condition in cities, whereas this approach, this nourishment, these studies, whether difficult or easy, are to 10
be gone through in this way. But it is not lawful to neglect the gods, once the well-grounded utterance about them all, proclaimed in the right way, has become manifest. But he who has accepted all these things in this way, he, I say, is truly the most wise. For nobody may hope to gain with ease the fairest and 15
most divine nature that god has granted that man should see without these things which have now been stated.

In addition the individual[61] and specific should be added in each of the forms of mathematics until we discover the whole cosmos[62] which the most 20
divine reason arranged in visible form. This he who is blessed by fortune[63] first wondered at and then acquired a desire to understand so far as is possible to human nature, believing that he would thus best and most fortunately lead his 22,1
life, and after death would arrive at places fitted to moral excellence, and, being initiated truly and fully and having, as one unified, gained a share in the unified wisdom, would spend the rest of time as a witness of the fairest things that 5
might be seen.

Also one should draw together the various uncoordinated subjects of study that comprise the educational system into a synoptic view of the interrelationship of all mathematical sciences and the nature of being; for only

such learning is stable for those who acquire it. One must also become capable of freeing oneself from [dependence on] one's eyes and other senses, and proceed towards being itself with truth. Also one must be steadfast in these studies and keen, as well as possessing such other characteristics as are fitting to the best nature. For if we educate by bringing men sound in body and mind to so great a study and so great a discipline, justice herself will not reproach us and we shall preserve the state and constitution. But if we lead a different kind of person to these things we shall fare in an opposite way and bring a flood of ridicule yet more upon the love of learning. For if one must tell the whole state of affairs as it is, in these studies this function of each man's soul that is destroyed and blinded by other ways of life is purified and rekindled – something that is more worth saving than myriads of eyes. For only by this is the truth seen.

23,1 To those who agree with this, these present things will seem to be incredibly well said. But those who have attained no insight into this will probably think that we are talking nonsense; for they do not see any gain worth mentioning from it. But it appears that this would not be the flipping of a potsherd,[64] but the conversion of a soul from a kind of twilight to what is a true approach to reality, and this we shall call a true love of learning. So surely one must investigate which branch of learning has a power of this sort, and which study will draw the soul away from becoming to being.

I say, then,[65] that some [observations] involving the senses do not encourage the intellect to inquiry, since they are sufficiently judged by perception, whereas others altogether direct it to investigation, since perception generates no sound result. Those do not encourage it which do not simultaneously eventuate in a contradictory perception, but those that do so I class as encouraging, since perception does not reveal the one rather than the opposite, whether it is received from near or from far. We shall understand better what I say if I put it this way: these, we say, are three fingers, the smallest, the next, and the middle one – consider me as intending them as seen from nearby. But please consider this about them: each of them appears to be equally a finger, I take it, and they are in that way in no way different, whether seen as being in the middle or at
24,1 the end, whether pale or dark, fat or thin, and so on, for all similar categories. In all these matters the soul of the many is not compelled to ask of reason what a finger is. For in no way did sight itself signal that the finger was the opposite

of a finger. So surely such a situation does not naturally call upon the aid of intelligence. But how about this: does sight sufficiently discern their greatness and smallness, and does it make no difference to it that one of them is situated in the middle or at the end? And similarly in the case of touch with respect to fatness or thinness or hardness. And do not the other senses exhibit comparable deficiencies in such instances? Rather, is it not the case that each of the senses acts as follows: first of all, the sense that is assigned to the hard is necessarily assigned to the soft, and informs the soul that it perceives the same thing as hard and soft? So is it not inevitable that the soul in such circumstances should be puzzled what the sense itself means by 'hard', if it also calls the same thing soft, and what does the sense of the light and the heavy mean by light and heavy, if it calls the heavy light and the light heavy? For these communications are puzzling to the soul and require investigation. So it is reasonable that in such circumstances the soul should attempt to investigate, calling calculation and intellection to its aid, whether each of the reports are of one or two things. For if they appear to be two, each appears as something different; but if each is one, but both are two, it will think of the two as separate, since otherwise it would not have thought of two separate things, but of one. Also, sight saw the great and the small, as we said, [but] not as separate, but as in some way mixed up together. To clarify this, thought was compelled to see the great and the small not as mixed up together but as distinct, contrary to sight. So then, it is from some such stimulus as this that it first occurs to us to ask what on earth the great and the small will be. And it is thus that we have come to call the one intelligible, the other perceptible.

This it is, then, that I have just been trying to explain, how some things provoke reasoning, while others do not; those which impinge on sense together with their opposites I defined as provocative, while those which do not are not stimulative of thought. Well now, it is easy to determine, on the basis of the above, to which group number and the one and other mathematical objects seem to belong. For if the one or any other of mathematical objects were adequately observed by sight or some other sense, it would not draw us towards being, as we discussed in the case of the finger. But if some opposite is always seen together with it, so that it appears no more one than the opposite, there will then already be a need for an evaluative element, and the soul would be compelled to raise the problem in itself and to investigate, activating thought in itself and enquiring what the one is, and

thus learning about the one will be among those things that stimulate it and convert it towards the contemplation of true being.

However, it is sight in particular that exhibits this feature in respect of the one; for we see the same thing at the same time as one and as unlimited in number. And surely, if this happens in the case of the one, the same experience befalls every number. Furthermore, reckoning and arithmetic[66] are altogether concerned with number, and these plainly lead towards truth. So it appears that this one will be to the highest degree among the studies for which we seek; the other branches of mathematics will also be similarly useful for understanding, since they lay hold on being but set us free from becoming; they call thought to their aid and stand apart from the senses, call us to the contemplation of the nature of being, and grant ease to the soul itself by its conversion from becoming to truth and being.

One should study mathematics also for the sake of knowledge; for the soul is in this way strongly led upwards, and it compels us to discourse about realities themselves, in no way tolerating it if someone, in discussion of them, brings to the fore visible or tangible bodies. For it speaks about those things about which reasoning alone is possible, it being not possible to deal with them in any other way. So mathematics is, it would seem, indispensable, since it can be seen to compel the soul to make use of intellect itself on the road to truth. Also, indeed, it makes people excel themselves in acuity, and further it provides much labour for him who studies and practises it.

One must also consider whether it tends to some degree towards making one discern the form of the good more easily. We say that everything tends in that direction which compels the soul to turn itself towards that place in which is the most blessed of what is, which it must by all means see. Surely it is suitable if it compels one to the vision of being; if of becoming, it is not. Also pursuits for the sake of knowledge are to be prized as studies if they are of eternal being and do not lay hold on what at various times comes to be and passes away. Surely these draw the soul towards truth, and form the philosophical mind towards gaining in the world above what now below we do not have as we should. For truth is seen only by such means as these. Therefore these things should be continually sought with concentration, in order that their character may become clear. For, other considerations apart, it is especially delightful and causes the soul to look upwards.

Such studies, then, are those which are concerned with being and the invisible, and such things as are attained by reason and reasoning, but not by sight. We should also use things visible as illustrations; however, one should not sedulously study them as if one would gain any truth from them about equals or doubles or any other proportion. For indeed it would be absurd to think that these things come to be always in the same way and never in any way change while bodily and visible, and to make every endeavour to gain the truth about them. For beyond all else one must be on guard against this, that those whom we educate should set out to learn anything pointless which does not always lead where everything should arrive. In that way their studies will be useful in the search for the fine and the good, but otherwise pursued they will be useless. In fact I believe also that this approach to all branches of mathematics, if it attains to [an understanding of] their community and kinship, and if it has worked out how these are akin to each other, brings their occupation to the things we wish it to and not to vain labours. For the liberation from bondage,[67] and being turned round from shadows to the statues and light, and the ascent from the cave and the perceptible to the sun and the good, and there the incapacity to look at animals and plants and the light of the sun, that is, at the pure forms and kinds, but at the divine apparitions in waters and shadows of realities instead of shadows of images cast by another such light as in judging shadows cast by the sun – all this business of the arts that we have gone through has this capacity and raises up the best element in the soul to the vision of the best among realities, just as in the simile the clearest element in the body to the vision of the brightest in the corporeal and visible[68] place.

Such, then, is the best use of mathematical studies and such is their supreme goal.

Chapter 7

What the special object of knowledge is which is the basis of each branch of mathematics, and how it is possible by division to make a general distinction of them, so as to know both the one and the manifold in mathematics, its character, and how it should be defined.

Since one must also, for each of the mathematical sciences, determine the proper object of knowledge that underlies each, let us distinguish, taking our start from a process of division, the species of studies with which they deal. In that way we may most easily understand the unity and the plurality of mathematical science, its character, and by what sort of differences it is distinguished. Let us, then, take our start from this point.[69]

29,1 The nature of the continuous and the discrete for[70] all that is, that is to say for the whole structure of the cosmos, may be conceived in two ways: there is the discrete through juxtaposition and through piling up, and the continuous through unification and through conjunction. Strictly speaking, continuity and unification should be called magnitude, being adjacent, and discrete plurality. In accordance with the essence of magnitude the cosmos would be conceived as one and would be called solid, spherical, and fused together, extended and conjoined; but, again, according to the form and concept of plurality the ordering, disposition, and joining together of the whole would be thought of as, we may say, being constructed of so many oppositions and similarities of elements, spheres, stars, kinds, animals, and plants. But in the case of the unified, division from the totality is without limit, while its increase is to a limited point; while conversely, in the case of plurality, increase is unlimited, but division limited. Naturally, indeed, and conceptually, both are unlimited and therefore indefinable by sciences; 'there will be no principle or object of understanding, since all things are unlimited', according to Philolaus.[71]

Since it is necessary that the nature of a science be discerned in things thus
30,1 given exactitude by divine providence, certain sciences have cut off part from each and limited their content; from plurality they developed the concept of quantity, which was already familiar, while from magnitude in the same way they developed that of extension. Both of these kinds they subsumed under sciences according to their own forms – under arithmetic quantity, under geometry extension. But, since these were not of a single form, each of them permitted still further subdivision. Thus of quantity part is absolute, freed from any relationship to something else, as for example the even, the odd, the whole, and the like; while some is relative in some way or other, which is termed relative quantity in the strict sense, such as the equal, unequal, a multiple, the superparticular, the superpartient,[72] and the similar; and again some quantity both is and is thought of as unchanging, other as changing and

moving. For this reason it is to be expected that two other sciences share in and are attached to the objects of each of the two above-mentioned sciences.[73] For in connection with arithmetic, to which specially belongs the study of quantity as such, music is allotted a role in the technical study of relative quantity; for its study of harmonics and concords professes nothing other than the classification of relations and ratios to each other of sounds, and the quantity of their relative height and depth; while for geometry, which is concerned with persistent and static extension, spherical astronomy arose as a collaborator and as arbiter of moving quantity, clearly of that which is most perfect and exhibits orderly and uniform motion. Therefore, since the sciences are themselves concerned with twin objects, it is sensible to think of these sciences as sisters, so as not foolishly to discount the saying of Archytas:[74] 'For these studies seem to be sisters', and it is sensible to consider them as connected together like links in a chain and joined by a common bond, in the words of the most divine Plato.[75] The uniform kinship of these studies should manifest itself to him who studies them in the right way; and him who has received all of them in the way that Plato himself proposes he calls the most truly wise, and playfully insists on it, and exhorts those eager to be philosophers that these studies, whether difficult or easy, should be pursued and chosen above all. This is entirely sensible, since a grasp of the continuous and the discrete comes about in these ways alone, while the cosmos and all that is therein contained consists of the continuous and the discrete. The accurate grasp of these is wisdom, and philosophy is the struggle for wisdom. But philosophy is the absolutely single one of all skills and sciences that is concerned with man's own natural end and leads to the state of flourishing which is proper to him alone among animals, and is naturally sought after as the goal most worth his seeking.

Chapter 8

What is the general criterion[76] of the whole of mathematics and how it is discovered from the division of the line[77] which the Pythagoreans hand down.

Next one must speak about the criterion for all mathematics,[78] of what kind it is and what differentiations of activities it contains within itself. So let us take

it from the beginning and by the application of division provide the complete doctrine about it.

Now all things intelligible are divided into two, those that are called specifically objects of intellect and knowledge, and those that are objects of discursive reasoning (*dianoia*).[79] The intelligibles are primary, and objects of discursive reasoning are secondary and inferior. Again, derived from these, there is another kind of thing, that consisting of objects of the senses, and of these some are specifically sensible and objects of belief (*doxasta*), others are objects of imagination (*eikasta*). The objects of belief that are sensibles in the strict sense are particular bodies such as rocks, pieces of wood, and the four elements;[80] these are primary among sensible objects, following on which there are others that are weak and not like them, but consequent on the primary objects; these are shadows, for shadows are consequent upon bodies, and they would not appear without some underlying body. For shadows, and also those in water and mirrors, are images whose being is dependent on other things and not self-subsistent. They do not appear without other things, but fall on other things, bodies, and if these are removed they do not appear. So they are generically sensible, since they fall under sense, but they are objects of imagination (*eikasta*) and of trust (*pista*) rather than substantial (*hypostata*). They are said to be objects of trust as among objects not demonstrable but rather accepted from the trust of those who proffer them. For after all things shadowy are not received of themselves, but from the bodies on which they fall and in which they seem to find a resting place. So such trust is insecure; for these objects, if isolated from the mirror or water or, in general, their foundation, would not exist at all. So, while bodies are objects of belief and have their being in seeming,[81] shadows are still further from reality by not having their own solidity but resting upon something else.

It is with these images that we may compare the objects of discursive thought, being in the same ratio to objects of knowledge and intellect as are imaged things to things sensible and objects of belief. For intellect possesses, as it were by contact, the forms which have really real being, but the objects of discursive reasoning that is to say, geometrical objects, are viewed by discursive reasoning, which does not any longer approach them directly and, as it were, by focusing on them,[82] but by reasoning rather than by direct approach to them. They descend, as it were, from the forms as if to copies and intelligible

images of them; analogously, things imaged and shadowy assume an inferior status to things sensed, since these themselves fall directly under the senses; while the former are viewed in and on and through something else. For clearly a shadow is not independent, but is either reflected on a surface, which is independently sensible, or in a mirror or in water, which are also independently sensible. In that way, then, mathematical objects also seem to appear, as it were, in the forms, and to rest upon them. For one should not think of them as derived by abstraction[83] from sensible objects but rather as descended from the forms and as having their character as images from them by having added to themselves <both plurality>[84] and magnitude, and by appearing in extension. For what in the images of sensible objects is weak and not self-supporting, that among intelligible things is what has bulk and extension. But, since this also strives to be without bulk and parts, it seems to rest upon the simplicity of the forms even as shadows do upon the firmness of sensible objects. 34,1

Accordingly, even as the objects of discursive reasoning are separated from intelligible objects, so also is discursive reasoning separated from intellect. For which reason Brotinus[85] also, in his treatise *On Intellect and Discursive Reasoning,* distinguishes them from each other in the following terms:

> The extent of discursive reasoning (*dianoia*) is greater than that of intellect, and that of the object of reasoning than that of intellect. For intellect is that which is simple and uncompounded, and intellect is primary, as is its object. Such is the character of the form; for it is simple, uncompounded and prior to other things. But discursive reasoning is complex and has parts and is intellectual on a secondary level, for it has added science and <demonstrative>[86] reasoning; and it is much the same with the objects of discursive reasoning, for these are the objects of science, are demonstrated, and are universals that are acquired by intellect by means of reasoning. 35,1

In this passage he says of thought and its objects that they are greater, meaning not that they are so in power but rather in plurality (these terms being understood as in opposition to each other).[87] He distinguishes them [*sc*. the objects of discursive reasoning] from intellect and its objects not only in these ways but also by the latter being simple and uncompounded, while the former have many forms and are compounds, and also because the latter intuit and are intuited primarily, the former secondarily and by receiving their actualization from them. Also the latter are embedded in forms, while the former are

actualized in reason-principles, which renders their activity multifarious. Also the latter are indivisible, the former divisible, the latter superior to demonstrative syllogistic, while the former develops syllogisms about existents. The latter themselves have being, while the former contain the particular in the universal and display it at the same time. Also the actualizations of the latter are immaterial and pure, while the intellection of the former is impure; for in its intellect it grasps its proper objects of knowledge by means of reasoning or by its intellect together with reasoning. The result of this is that the matters judged and their criteria of judgement are different from each other, so that the objects of discursive reasoning are different from those of intellect and discursive reasoning is different from intellection.

36,1 Archytas in his treatise *On Intellect and Sensation* still more clearly distinguishes the criterion of the various kinds of existent, and he sets out the most appropriate criterion for mathematical objects as follows:

> In ourselves there are through the soul four kinds of cognition; intellect, scientific knowledge, belief, and sensation. Of these two are bases for reasoning, viz. intellect and sensation, two are results, viz. science and belief. Like always has cognition of like; so it is plain that intellect in us cognizes things intelligible, science the objects of science, belief the objects of belief, sensation things sensible. Therefore discursive reasoning must move from things sensible to objects of belief, from objects of belief to those of science, and from these to things intelligible; <and these make consistent the truths which are perceived through them.>[88]
>
> Now that these distinctions have been made we must grasp what follows. For like a line cut in two into unequal segments and again segments cut off from each in the same ratio,[89] so also let the intelligible be divided in the same way from the visible and let each of the segments be divided in the same way and differ in clarity and obscurity from the others. In the same way, within the sensible the one segment is figures seen in water and in mirrors, the other part the things of which these are images – plants and
>
> 37,1 animals. Of the intelligible what is in the same ratio as the images are the kinds of mathematical objects. For those who, in the field of geometry, hypothesize the odd and even and figures and the three kinds of angles construct the rest from these, and they leave aside the question of the nature of the entities themselves as if they were thoroughly cognizant of this, even though they cannot give an account of them either to themselves or to others. Rather, they use sensible objects, as one would expect, but they do not

inquire into them and do not construct their arguments with reference to them, but with the diameter and the square itself in mind. But the other section is of the objects of intellect, which is the subject-matter of dialectic. For the intellect in fact makes the hypotheses not principles[90] but starting points and stepping stones until it comes to the unhypothesized supreme principle, and in turn it descends to the end without the use of anything sensible, but forms themselves and of themselves.

It is a good procedure, then, to divide the states of the soul into these four segments, and to call that in the highest one intellect, that in the second discursive reasoning, that in the third trust, and that in the fourth imagination.

I think, then, that through the above also it has become abundantly clear that there are four distinctions between levels of reality and four principles (*arkhai*) of judgement; and as reason, being in a central position, adjoins the two extremes – the objects of intellect and of sensation – it fills the role of end or purpose (*telos*) with regard to intellect and sensation, since they are its own starting points and it is completed by them.[91] There is also this common postulate with regard to every cognitive capacity, that like is known by like. So it is possible to come to understand each pair from the other and each one of these from its counterpart, and to make equal divisions both in common and individually concerning them, and to make an order of transitions from the ones to the others, i.e. from the inferior to the superior. Also he made clear how one must make an ascending order of everything towards the intellect. 38,1

After that he divides the line, which is a single continuous one, so that we may understand the faculty of cognition as being one. But he divides this in accordance with the primary differentiae between things [*sc*. the intelligible and the sensible] and the judgements (*kriseis*) about them, that are also divided into two. He assigns to them [*sc*. the two sections of the cognitive faculty] an equal status through the participation in reason-principles and forms and because of the similarity of those that participate and those in which they participate, and because there is in a way the same relationship within both pairs. He again divides each of the segments in the same ratio, since throughout the cognitive faculty is internally of the same kind. He makes the differences in judgement to consist in clarity and obscurity and complete or incomplete determination[92] and also points out the distinction between them according as the second pair varies from and is inferior to the former.

39,1 First he divides the sensible, as being more familiar, and takes from it the level of being that appears as an image derived from sensible things, images in water and in mirrors, cutting them off as being of some one nature. Then he separates 5 off the other part, which is the genuine one, of which the others are images, such as plants and animals; for it is from these that there comes about imaging (*eikasia*) of images by the reversion of sensation into these through reflection, and thus cognizing them in a secondary way, just as they exist in a secondary mode. But 10 the senses fall upon sensible things immediately, as they exist primarily and have in themselves a material level of being. It is possible to understand the other segment also from the analogy with these. For the kinds of things involved in 15 mathematics exhibit an analogy with images, and cognition of them has some similarity to the imaging of images. For they are actualized from intellection, and people move from objects of intellect to mathematical objects as to images, and they rely on hypotheses and do not know the causal principle. So that is the 20 criterion of mathematical objects,[93] that it is cognizant[94] of something other and not the object of intellect, and also that it seizes on the object of discursive reasoning by another form of cognition, and not by intellection. For this [intellection][95] is the criterion of the dialectician, and by that he both views true 25 being, forms, and all things not based on hypotheses,[96] and also can give an account of all things and make use of nothing sensible, but of the intelligible 40,1 forms. So then, as there are four faculties of judgement, an order is to be observed and their operations distinguished as follows: intellect of the highest, discursive reasoning of the second, trust of the third, and imagination of the fourth.[97]

5 From these considerations, then, it has become sufficiently clear by division what is the criterion of mathematical science.

Chapter 9

About those that propose a definition of the essence of mathematics, of whom the first viewpoint is presented as being that of those who relate it to the soul; several causes of such a conception are stated, and starting points for the whole study of mathematics.

But if one is next also to provide a precise and comprehensive account of what the form of mathematics is and what its status, let us first observe the opinion

of those[98] who assign it to the soul; for one could bring reasoning to bear upon this in a precise way.[99]

Now it would not be reasonable, by such a focus[100] on the study, to make the soul one kind among the objects of mathematics; for in that way knowledge about mathematical existence would become divided. Therefore with regard to the soul one should not produce a definition of it as either the form of the totally extended[101] or a self-changing number,[102] nor a harmony consisting in ratios,[103] nor anything else of that sort on its own, but all should be woven together, taking the soul both to be a numerical form and to consist in numbers containing a harmony. And all kinds of symmetry in common such as fall under mathematical science should be assigned to the soul, and all ratios together should be subsumed under it. For this reason it subsists as the common product of geometry and arithmetic and harmonic ratio, and hence it is the same as proportionate ratios; also it has a certain kinship with the principles of true being, is linked to all levels of being, and is capable of being likened to them all.

Such then are the grounds of such a conception. We would obtain starting points towards the study both of mathematics and of the soul, if we were to see how all that is limited[104] and determinate reaches the soul from numbers, and how its unitary account does so from the nature of the unit, and how the power which proceeds into magnitude and increase and has a superfluity, so that it can give it to all things fitting, comes to it from geometrical being. But the power of harmonized change, order, and symmetry of irrationals and good measurement in numbers commensurate[105] or including the commensurate come from the essential power of harmony. Therefore the soul both listens to harmonies and rejoices in things that are harmonized, as being itself a harmony, and it acquires its being from numbers and all other such mathematical measures, which have assumed a kinship both to the intelligible forms and to objects of sense and enmattered forms; so the present doctrine provides a starting point for study of all these, on the ground that such a proposed conception of mathematics provides all such concepts.

But to draw together the whole theory, we conceive of the soul as having its being in the common reason-principles of all branches of mathematics, as on the one hand having the ability to judge them (*to kritikon*) and on the other as being responsible for the creation and construction of the incorporeal

measures with which one may harmonize the creative activity of the enmattered forms and their production through images, which proceeds from the invisible to the visible and joins together things outside to things within. In a word, in all these respects the account of the soul embraces from itself the whole development of the mathematical sciences.

Chapter 10

How the essence (ousia) of the soul is composed out of all the mathematical sciences, and according to what distinction the mixture of them within it may be determined, and whether the soul encompasses within itself the whole level of being of mathematics, or if some other principle of mathematics is also to be considered.[106]

It is important to investigate thoroughly whether the soul is a mixture of all the varieties of mathematical entity, or rather constitutes them all according to a single primary structuring principle (*logos*). If, on the one hand, it is a mingling of the whole of them, the constituents from the coming together of which it is constituted will exist prior to it, and it could no longer be the source of the essence of mathematics, but rather the soul will be an offspring of the congress of randomly existing branches of mathematics,[107] in addition to other absurdities, and it contrives a composition with the soul and a posterior level of being, since it comes into existence from prior constituents.

If, however, the soul is the primary source of mathematical existence, and it produces this from itself, it will be senior to it, it has precedence as a cause, and is superior as a distinct entity. This, however, would run somewhat contrary to the present line of argument, for the soul will thus become more estimable than the constituents of mathematics. So it is better to say that it neither takes precedence over mathematical objects nor follows upon them, but runs together with them and exists together with them, containing an uncompounded and indivisible mixture both inhering in the whole and emanating from the whole of them, present with them in a single form, sharing in the whole as a unity, containing in itself a power to hold the whole together, and likewise having given itself to the whole of mathematics. If that is the case, the soul both encompasses all in itself, completely and self-sufficiently, and it leaves nothing

outside of itself (for it is itself complete and it is not possible for anything that there is to be omitted from its own principle). Thus its essence will be single, as there is just one and the same principle pervading the whole. There will, however, nonetheless, be differences according to the different powers, forms of life and activities of the soul and the variety of its forms of being, albeit contained in a unitary structure.[108] Some such specificatory distinction, then, one might reasonably propose on this topic.

And so let that much be our account of the essence (*ousia*) of mathematics.

Chapter 11

What is the function of the study of mathematics, and how it arises, and that it is called mathematics conformably to these.[109]

The function (*ergon*) of this science is not well defined, nor is it always concerned with the same matters in the same way, as is the function of the intellect. Nor is cognition in its field entirely derivative from itself, in the way that for intellect such a characteristic is innate. It is stimulated externally to its acquisition of knowledge;[110] it receives the principle of its recollection[111] from outside, and it then projects it from itself. Also, its function is not fixed in a single activity like that of intellect, but rather by changing proceeds from and to itself. But also it is not complete in itself, as is the intellectual, but it always advances by search and discovery from some gap in its knowledge to its filling. Likewise, it is caught in the middle between limit and the unlimited, whence it is always moving forward from the unlimited to becoming determinate, and transports itself towards sharing in the mathematical forms.

For all these reasons this science is acquired from prior learning; the basis for this the teacher provides, and then discovery follows, and this is dependent upon the first principles laid down by the teacher. For through these the soul recollects the true forms in mathematics and formulates their appropriate accounts. Sometimes, indeed, the activity arises in common from both, which is why Archytas says in *On the Mathematical Sciences*:

> For one must acquire knowledge either by learning it from another or by discovering oneself things that one did not know. So one thing is learnt from

15 another and it is not one's own, another thing is discovered by oneself and it is one's own. But to discover without investigation is difficult and uncommon, while by investigation it is abundant and easy; but without understanding investigation is impossible.

In this passage, then, he set learning first as the starting point of such knowledge, 20 and he indicated what is special about it, that it is provided by another. He added 'discovery by oneself' as coming second. For even if that is prior in terms of potency, still in human order it comes second relative to us; for inevitably those who have fallen into becoming must first be summoned to reminiscence by others.

25 So we can conceive of these ways of acquiring knowledge as two, but it is 45,1 also possible to include them together in our calculation as one. For when we participate in mathematics as received from another, and not from our own initiative, we then make it our own as coming from ourselves. It is easy to 5 grasp this from the observation of discoveries; for we discover and reflect on the things discovered as if having them within ourselves. Also the same becomes clear from investigations; for, if it is impossible to investigate without knowledge,[112] there was a time when we knew these things, and this is not the 10 present time, since we are currently ignorant of the matters in question; therefore we knew them previously. For this reason mathematical discoveries are abundant and easy if one initiates an investigation, but difficult and rare for one who does not investigate. This is because they are present in a way in souls and were there formerly for souls as active knowledge. So the discipline of 15 mathematics is a road from investigation to discovery and from learning (*mathêsis*) to investigation and discovery. From this it got its name and is called mathematics; for the science got its name from that from which it receives its 20 beginning, without which it cannot be acquired, I mean from learning; and from that it got its name.

Let these points, then, be included thus in our elucidation of the science.

Chapter 12

What are the capacities of mathematical science, the orders within them, the number of differences by which they are divided, and in how many ways they are conceived.

One might enumerate as well a whole range of capacities (*dunameis*) of mathematics, but among the foremost should be observed those that combine from multiplicity to the same unified cause and those that divide into multiplicity from unity. For since mathematics is composed from limit and the unlimited and from the one and the many, and is intermediate between the complex and the simple, it also, according to the same account of its being, partakes of both collection and division, both expansion and compression, and both reversion to the definite and departure from it. A similar contemplative activity is to be found also in dialectic, which especially deals with being, though it possesses not a small difference from the former. For the latter contemplates pure being and collects or divides it, but the former controls the mathematical and is active in reasoning about it in these two ways.

There are also other capacities which consider what is common in many things, such as survey in the different branches of mathematics certain common forms, common ways of reasoning, and common measures by which different kinds are distinguished, such as the theories of equality and inequality and of symmetry and asymmetry. For these capacities survey features common to many areas.

Opposed to these are those that study the special character of each branch, of numbers qua numbers and magnitudes qua magnitudes, and so on in other cases. Among such are also those which have an intuition of the principle of analogy pervading all things, beginning from above with the most primary things and finishing with the furthest removed, proceeding through the intermediate, observing everywhere the same and the different accounts of different things, and making their presence clear at all levels. Not but what those capacities also which survey the beauty and the measure of mathematical entities, as also their harmony and symmetry, should be provided with some account; for these have through their own nature good order and completeness and every merit which mathematical forms should have.

Now to the many these powers seem to be unchanging and to activate knowledge about unchanging reality. But this assumption is incorrect; for there are some branches of mathematics whose province is the numbering of change[113] and its measures both absolutely and relative to each other, with regard to their ordering and symmetry. The incorporeal cycles of the soul to which the circular motions of the heavens are conjoined are also in their

province, both how they acquire their symmetry and according to which numbers, and why they are interconnected, and all similar matters. In these branches of mathematics both astronomy and harmonics are included. Therefore, in order that the capacities (*dunameis*) of mathematics should extend also to those branches that study motions, these also must be assigned to the sphere of mathematics. But we must distinguish from them those that are static and which study unchanging forms and reason-principles (*logoi*) and are able to reason out their relative positions – among which are included the majority of the intrinsic capacities of mathematics. Their orderings with regard to the character of the objects of knowledge which they study must be distinguished, if some are pre-eminent and others subordinate, and also their order with regard to the importance they give to beauty, if some study the superior and supreme beauty, others that which is inferior and incomplete. The differences between them must be gathered from the character of their activities, from the differences of their knowledge and from the different entities associated with them, with regard to which they construct the sciences relative to each. In how many ways they work is to be investigated from the changing objects of knowledge of the science; for it is from these that it can be seen in how many ways the capacities themselves are instantiated and how many forms of activity they engage in.

This, then, is how one may set out these matters in a preliminary sketch. But one must, as is proper, wait for the complete account about them; for it is in that way that most comprehensive instruction about them may be given.

Chapter 13

What elements and kinds (genê) of mathematical science there are, and how the same ones are present as elements and how as kinds; by what these are different from those in the other sciences and kinds of things, both the intelligible and such as are in the realm of generation.

Every study and every science receives its unchanging character from its primary elements (*stoikheia*), when these are definite and never varying, and it provides the most complete comprehension from a progress through the

elements. Also each of them discovers the mode of reasoning and proofs suited to it on the basis of the kinds (*genê*) befitting it. Accordingly it is necessary also in mathematics first to separate out its elements that are most common to the whole structure of the science, and to make a hunt for such kinds as are most central to it and extend furthest to all in common. When we see these things, we may in turn examine whether the elements and the kinds will be different from one another, or whether the same things may be studied in one way as kinds and in another as elements, and how such things are different in mathematics from the kinds and elements in other sciences and substances, both such as are on the intelligible plane and such as are involved with becoming.

The best of those engaged in philosophy are agreed that the elements and kinds of objects in mathematics are definite and ever unchanging, and mathematical demonstrations themselves clearly confirm this, since they remain always unchanging. It is easy to comprehend that the account of these also is in accord with the first principles of mathematical substance. For in their case also the one and plurality, limit and the unlimited, identity and difference are elements and kinds of the science and of the matters that are learnt within its ambit. So, when we view these as causes and creators of the whole being of mathematics and its study, the causes now mentioned must be conceived of as principles; but when they are conceived of as part of and constituents in the being and account of the science, they are to be thought of as elements in it. So when we view them as common to all mathematical sciences, providing a connection within the being of the parts of them and nonetheless existing in their own right, then we may see them as kinds. Thus these same things are in a way the initiators of the study of mathematics and the objects discerned as existing within it, while in another way they are conceived of as elements, and in another way again as kinds of subject-matter. This is not because they are different only notionally and as a matter of terminology, nor because they are subject to mutual change and become one sort of thing after another according to their different relations, but rather because the same things exhibit many progressions and internal differences. For indeed they remain undifferentiated with regard to difference of cause; for it is not the same thing to be an incorporeal entity as such and to be a component of other things, but nor does being creative of other things and in

50, 1 some way contributing to their very own being[114] merit the same status. Nor does what is combined with other things resign its very own being to the 5 complex, but it sets that apart and weaves in its other being towards the completion of those other substances to the being (*einai*) of which it contributes. For the same reason, nor does the cause which bestows being join its essence to the things it brings into being, but is senior to them according to 10 the account of its being, and it has separate within it its level of being[115] through which it gives another level of being from itself also to the totally separate things derived from it.

So in that way we may reasonably assign the unlimited and the limit among both principles, and elements and kinds of objects. But these differ from 15 intelligible principles, elements and kinds because they fall below them in completeness, clarity, and simplicity, as also in greatness of content both in extent and in beauty and in all things good. On the other hand, they are superior to things in the realm of becoming in order, in symmetry, in 20 unchanging and stable nature, in pure participation in forms, in incorporeal and immaterial nature and, in a word, in everything that is better. From this it follows that they are intermediate between these two and have a central position which can engage with both and act equally as a bridge to both.

So then, if one recognizes such distinctions about these matters, one would not fail of a fitting conclusion.

Chapter 14

Concerning the similarities and dissimilarities in mathematics, what they are and how far they extend; how they are present in the subject-matter of mathematics, and in what respect they differ from similarly named kinds, both of intelligible and of sensible objects.

With regard to similarity and dissimilarity, everyone would agree that there is 51,1 much of this in mathematics and within the mathematical realm of existence, and that it has there a great influence. For it is not possible to understand a mathematical theorem unless one makes a construction of it by determining 5 some figure like it, by carrying out one's reasoning about it by means of some

image of it, and by constructing one thing from another according to one rule, that of similarity.[116] But this is worthy of consideration, what these common characteristics of the similar and the dissimilar are, how far they extend in mathematics and what their place is there, and in what respect they differ from features of the intelligible and sensible realms that bear the same titles of similar and dissimilar.[117] One must, however, realize this, that similarity is not predicated qualitatively in the case of mathematical existence, nor does it supervene as one thing on others in the category of figure, as in some contexts.[118] For such qualities tend to occur among composite things and in the context of composition, in which cases the substance is one thing, and what is incidental to the substance, providing a character and structure for the basic nature, is another.

But the similarity and dissimilarity which we are now looking for is superior to every composition, nor is it viewed relatively, in the same way as in the category of position. For one should not conceive of the existence of things that exist in their own right as dependent on others. So let the similarity and dissimilarity now being discussed be primarily of substance[119] – not, however, of all substance, but of the mathematical. For these similarities and dissimilarities will be forms of mathematical substance.[120] For one must not distinguish quality and quantity off from substance, and assign one science to the study of substance, another to that of quantity, and mark this off as mathematics; rather one should investigate, in accordance with its nature, the proper being of mathematics, and how many species there are and what they are like; also that which at the same time examines quantity, but not that in bodies, nor the intelligible paradigm, but such as is mathematical.

In the same way, then, it is its natural function to survey at the same time similarity and dissimilarity with regard to the same account of their being, whether as common genera related to substance, or species thereof, or as common powers giving birth to such species as occur in the particular mathematical sciences. They extend in the case of mathematical substance into the whole of its being, in the case of the special sciences to their separate levels of being, and thus, so far as one surveys more or fewer, greater or more impoverished outlines of them, similarity and dissimilarity are coextensive with them all, reaching out to a greater or lesser extent. For altogether neither plurality nor division nor unification nor identity and difference can exist

among genuinely mathematical objects without the precedence of substantial similarity and difference.

So let us no longer wonder if they extend to a single genus, to more, to fewer and to all, but one should much rather observe this point, how they are present in them according to what is their own by kinship in each case.[121] And one should observe this also, I mean the unit, the many and the things intermediate between them with regard to similarity and dissimilarity, and one should note what is their fitting order, and, in the case of each of the sciences, what is the natural share of each of them in similarity and dissimilarity.

Now if this is how these matters stand, then mathematical science must be supreme among all of them, since it discovers the same cause of similarity and dissimilarity in the case of prior and posterior objects alike, and, in the case of things of equal rank, makes the appropriate distinction between them. And moreover, contrary to the general opinion, corresponding to the character of incorporeal things, it observes the similarity between differing things and the dissimilarity of those without difference. Likewise it searches out the areas of opposition within each other, even as it discerns dissimilarity in the similar. So we should best recognize how each of them is present if we were to take into account here also the special feature of the substance of which we are now treating. For we should not think of them as being present in the way that enmattered forms take visible shape in conjunction with matter; for they are organically fused with and inseparable from mathematical substance, in which they have their being; but nor should we think of them as what is organically present in bodies as is heat in fire; for even if these also are totally coexistent with the things that possess them, still some difference within the composite is observed in so far as they are possessed as something else, but bodies possess them as something other.

But in the case of those <similarities and dissimilarities> which are in their being pre-existent in mathematical objects a simple substance is observed which is throughout without composition. For the more the mathematical object is incorporeal and separate from composite and extended masses, so much the more does it have a simpler, self-subsistent and purer unmixed similarity and dissimilarity. Similarly what is common to them is crystal clear and free from contamination.

So when we reach agreement on these points we must also learn this, that mathematical similarity and dissimilarity must be conceived of as different

from the similarity and dissimilarity of things intelligible and sensible. It will be distinguished from them, then, in one way, that by which we separated out the three kinds of entity; for it is clear that since these are different in three ways their forms will also differ in three ways. It will be distinguished in another way because it is marked off by its intermediate position, while the others are extremes and mark off the beginning and the end of the forms involved in their own proper levels of existence; also in another way one lot should be thought of as associated with originative principles, others occur at the level of outcomes, while yet others come naturally in between the primary causes and the final results. If we first accept these distinctions, we should be able easily on this basis to observe the special similarities and dissimilarities in each branch of mathematics whenever we are treating of it in particular. But for the present let our account of the common features end here.

Since, then,[122] mathematical science consists of such and so many kinds of principles and elements, and has the character which we have already described, its extent is by no means confined to and concerned with just a few of the affairs of life, but contributes to the most important and best of divine and human goods.

Chapter 15

How the whole of mathematical science extends, both itself, its kinds, its elements, and its principles, into the whole of philosophy and all the parts of philosophy; how it is involved with them and by what sort of contribution.

Now[123] first let us venture to state that mathematics extends through the whole of philosophy and every study within it of both things in being and those in becoming. In fact, the whole of it and its branches, its elements and its principles, and all that are genera or species of mathematics, permeate the whole of philosophy. That is indeed why men use mathematics everywhere when engaged in any philosophical study. For since mathematical objects are incorporeal and intermediate and thus are able to fit with all kinds of things and become similar to them, they make a great contribution to all philosophical sciences.

For[124] mathematics prepares a suitable apparatus for theology and a receptivity for it, and provides an assimilation to it and an induction and a cleansing away [of alien elements]; it frees its intellectual instruments from their chains, cleansing them and uniting them to being. By the beauty and order of the theorems of mathematics it approximates in a way to things intelligible, and through its study of the invariant and unchanging it is likened to the stable, constant and well-defined objects of intellect. It gently accustoms the reasoning faculty to focus on the radiance of being and leads it away from bodies, it induces a confidence in the being of things incorporeal, and it provides scientific certainty and accuracy. For all such things provide a great impetus to the recognition of intelligible beings.

However, it is no mean tool of the natural scientist, since it indicates the symmetry of natural objects, the great extent of their good order, and the analogy which extends throughout everything in nature. It surveys their beauty, their natural kinds and the reason-principles relative to them, their most simple elements and their figures, and the most important genera and species. For all this is the usage of those who genuinely study nature from first principles. It is a help to the statesman by prescribing orderly patterns of change in the sphere of actions, infusing motion into static theorems,[125] and providing fitting equality and agreement for all. For the moralist it helps in providing accounts and paradigms of excellences by revealing mathematical forms, such as of friendship, or happiness, or some other of the greatest goods.[126] It sets out mathematical paradigms of all the constituents of life, such as fruitfulness and barrenness, fertility and infertility, and all such things. So for this reason one must make use of mathematics everywhere, providing a general outline, as it were, of philosophy with these paradigms. So we do not use the same paradigms in all circumstances, but we set out appropriate ones in each case in accordance with the specific branches of science.

Thus both the whole essence of mathematics itself and its branches, its elements and all its principles extend throughout all philosophy. For it is possible to extend mathematical procedures (*logoi*) generally throughout the whole extent of philosophy, and it is possible to extend it also throughout the parts of philosophy themselves as the needs of reason demand. It shares in them in so far as it has some similarity to them, and it provides some contribution which transports and guides one to them. It is both of a nature to

raise and lead us to the unchanging and definite forms which do not at some times exist, at others not, but remain always the same; and while it is inferior to them in perfection, clarity, and the fineness, so to speak, of incorporeality, it is similar to them as to superior entities. It also sets out in mathematical forms separate from bodies paradigms of the enmattered forms of things in becoming. 57,1 5

Thus mathematics provides help of this kind to philosophy as a whole and to its parts.

Chapter 16

How many goods it provides to the arts and crafts, universally to them as a whole and to their different kinds, as it does to the theoretical, creative, and practical crafts; and succinct instruction about them.[127]

Mathematics injects a scientific understanding into more or less all the arts and crafts *(tekhnai)*.[128] It indicates their principles, their ends, and their definitions; it teaches their bounds and determinations; it discriminates between their correct and incorrect procedures and marks off the appropriate elements of each of them, recognizing their proper purpose (*telos*), providing accuracy [in their delineation], and indeed effecting their discovery [in the first place]. For since this science studies essence apart from matter and uses reason-principles (*logoi*) which are separate and not put into disarray by enmattered things, for these reasons it has, as one would expect, a more causal and guiding role than the arts which deal with matter, in respect of their discovery, evaluation, and distinguishing one from another. 10 15 20

So it purifies and perfects the contemplative arts, it is present in the role of a pattern for creative arts, and it stirs up and sets in motion practical arts by means of its unchanging forms; while for all of them in common it harmonizes their separate reasonings (*logoi)* with the aid of its enmattered forms. As being supreme over all it is their leader, it takes charge of them, makes itself of use to them, and makes them valuable and useful by stabilizing them through the imposition of mathematical proportion (*logos*), and it controls their reasonings and makes them accurate by the application of mathematical demonstration. 25 58,1

Thus mathematical theory is revealed as pervading all work and knowledge connected with the arts. 5

Chapter 17

What the order of presentation in mathematics is, and if it has a natural order also for learning; and if each ordering is appropriate to each, and the two to each other.[129]

Hence it is easy to recognize that there is a twofold ordering in mathematics, one of them conjoined with its very nature, the other with regard to learning it. For if their assigned order accrues to all other [disciplines] from mathematical science, and the necessary sequence of one aspect on another, then surely mathematical study contains in itself a much prior ordering, and it makes its progress towards perfection in orderly fashion. So the natural order in mathematical sciences assigns a prior place to the simpler, as for example to number theory (*arithmêtikê*) before geometry, and sometimes the same things have priority for instruction, when learning of complex doctrines arises from the elements. However, sometimes from our perspective the complex will be prior to the simple in learning, as for example the whole heaven and motion round it is obviously more familiar than the sphere in the abstract and the motion of the sphere as such.

But if one were to demonstrate the obscure from the obvious, such a way of approach is not to be discarded. So once these methods of procedure have been distinguished, one should make use of both of them, the one as more
59, 1 scientific, the other as more familiar. So when it is necessary to use the one method only, one must prefer the more appropriate of them, which contributes more to the scientific subject before one. But when it is possible to use both, one should lead people to knowledge by both procedures. Hence in many mathematical studies the same problems are demonstrated both by analysis and by synthesis.[130] So where the two modes of knowledge are complementary, in such cases both should be used. But one must size up the character of each student; for example whether, being clever and keen, he can proceed easily from one to many and accept many things all together when they have some affinity with each other. Another thing to be considered also is what the aim of engaging in mathematics is, whether it is simply learning the theorems of the science or whether someone is connecting them to philosophy and proposes to make his way through them to the vision of the intelligible. For such a one

there should be a different order of study, which would sometimes overrule the natural mathematical order.

Again, each mathematical theorem indicates that some things that are plain and less developed come first, such as that the right-angled triangle has the square on its hypotenuse equal to those on the adjacent sides; but other things which are more developed and requiring complex proof are taught later; for example such truths about the right-angled triangle lead on to the motion of the stars, to the complete comprehension of the zodiac and the motion of the sun and the moon. Similarly in matters of harmony, facts about harmony in general are taught first, those about the harmony of the universe later.

We have provided these prefatory remarks in order that in using some method for ordering work in mathematics we might accommodate two things, the nature of the subject-matter and the capacity of the students, and may use each method suitably; and that, when these methods are complementary to each other, we should use both indifferently.

Chapter 18

The special methods of the Pythagorean presentation of the mathematical sciences, how and in what matters they made use of them, and for whom; also that they always took due account both of the subject-matter and of learners.

Indeed the special methods of Pythagorean teaching of mathematics were marvellously accurate and greatly surpassed the skill of those who were engaged in the teaching of mathematics on the technical level. So let us present an outline of it, so far as is possible to speak about it in general terms.

So let this one thing be agreed, that by starting from above from first principles they provided the first structuring of mathematical theorems, undertaking their reasonings on the basis, as it were, of the theorems' primal being itself, and finally leading the whole mathematical enterprise back up to this focus.[131] Further, as a consequence of this, they were accustomed to exhibit first the discoveries of the theorems and to take nothing for granted, but in all cases to show how what was being demonstrated in mathematics came to be the case. They also shared another procedure, the mathematical use of symbols, e.g.

the pentad as a symbol of justice, because it signifies symbolically all the forms of justice.[132] This form was useful to them in all philosophy, since they performed the majority of their teaching through symbols, and also believed that this procedure was appropriate to the gods and befitted nature. But, indeed, it is clear from the other mathematical sciences that they provided the first principles and discoveries of mathematics, and it is plain also from their methods in number theory. For they teach clearly how each kind and species of number first comes to be and how it is discovered by us, on the principle that the study of numbers was not scientific unless one should grasp them by starting from above.

Furthermore, they always assimilated the theorems of mathematics to true beings[133] and everything divine, to the conditions and powers of the soul, to the heavenly phenomena and the orbits of the stars, to all the elements of bodies in becoming and the things composed of them, and to matter and the things that come to be from it, likening mathematical theorems both in general, and taking from each its own points of similarity to each part of true being. They applied mathematics to things either through their having the same reason-principles (*logoi*) in common or through some dim impression, or through a likeness either close or distant, or through some association of images, or through some dominant cause as in the role of a paradigm, or in some other way. They also join mathematics together with things in many other ways, since things can be likened to mathematics and also mathematics has a nature to be likened to things, and both can be assimilated to each other.

So they were not at all pleased by the elaboration of terminology and the wealth of procedures, since it was rather too logic-chopping and divorced from the true nature of the facts, but they particularly welcomed recognition of the problems themselves, as contributing to the knowledge and discovery of reality. They emphasized the discovery of truths, and focus[134] on realities rather than the shrewdness and acuity of reasonings about the facts. Therefore they did not set a high value on dialectical argumentation in mathematics, but they evidently esteemed more highly that which contributed to the discovery of realities.

So these methods and the like were those they used for mathematical teaching. But they used them scientifically, in conjunction with the theoretical philosophy of true beings, and aiming at the Good. For they thought that they ought always to give precedence to and honour the limited and the most succinct, while selecting from these whatever was useful for themselves and

their companions and for complete knowledge of true beings. Also, however, in their instruction they aimed in one way at the facts, their ordering and their mutual relationships; for they marked off their first and second theorems with reference to such a sequence; but in another way they also turned their attention to their pupils, their capacity and how they could be helped by them, and to what should be taught to beginners and what to the more advanced, and what sciences were esoteric and what exoteric, which could be spoken of and which could not, and to whom instruction should be communicated with scientific understanding of the facts and to whom only mathematically. For accuracy in all these matters was pursued by them not without purpose, but in order that mathematical study should hold on to one thing, the fine and good, and should be aimed at one thing, knowledge of reality and assimilation to the Good itself. So then, in this way not only a bare knowledge of mathematics was handed down, but also a suitable way of life was joined to it, and an ascent to the most honourable ends was suitably provided by it. Therefore it is worthwhile to occupy oneself in accordance with the Pythagorean practice in mathematics, as excelling and preferable to all other mathematical skills.

Chapter 19

The Pythagorean division of mathematical science as a whole into genera and the most important species, which creates a common study of them.

But since we must survey not only the merit of mathematics as a whole but also its kinds and species, their number and which should be chosen, let us make our teaching about them general and capable of extending both to the whole and equally to each branch of mathematics.

The primary study of every mathematician, including the private individual, in each matter, whatsoever it be, is the theological, by harmonizing it in rank and activities with the being and the power of the gods by some fitting analogy. This, indeed, is thought to deserve the greatest concern from men, as for example, in the case of numbers, which sorts of numbers are akin to which sorts of gods and of like nature with them, and in the cases of other branches of mathematics they are accustomed to have the same conception. Now after

64,1 that among these men mathematics endeavours to exercise itself concerning truly real and intelligible being, the intelligible circle and the formal number, and they pursue other similar branches of mathematics in conformity with purest being. Again they bring the practice of mathematics under the same 5 head as the study of self-moving being and the eternal reason-principles by defining that same self-moving number[135] and by discovering certain measures of ratios (*logoi*) through certain mathematical symmetries.

Much mathematical science also studies the heavens and all the heavenly 10 orbits, both the fixed and those of the planets; it also works out not only the complicated motions of the spheres but also those of them that are uniform. It also concerns itself already with reason-principles in matter and enmattered 15 forms, what they are like and how they were initially brought about; for such is that part of mathematics which separates conceptually form and figures from bodies. Further it endeavours to give an account, in accord with principles of natural philosophy, of things in the world of becoming, studying both the simple elements and the reason-principles concerned with bodies.

20 So the Pythagorean school uses all these parts of their procedure in each and every branch of mathematics, and by them produces order and purification.[136] For as in mathematics second-level principles are made known 25 from more basic ones, so in the case of the powers of the soul the ascent to more complete lives and activities comes about through mathematics. However, they neither neglect nor omit anything intermediate which goes to complete such a science nor leave unexplored the extremes. They traverse the whole 65,1 without omission, and thus this science hands down, in the case of the most important and primary topics, that division which the science of division demonstrated. From this division it is possible to find out also the subdivisions 5 of mathematics, of which we shall make mention as we proceed in the discussion (*logos*) specifically devoted to this.[137]

Chapter 20

What is the method of definition in mathematics and how it comes about; the kind of help it contributes to science; that mathematics has a goal and what sort of goal it has.

However, there is another capacity of mathematics complementary to this, that of definition; for mathematics makes use also of definitions, and frames them with exactitude. The character of the whole construction (*sustasis*)[138] of definition is as follows: when the divisive process of mathematics divides its subject-matter into genera and species, then the definitional process collects together the differentiae resulting from the division and gathers together a single common account from them all. It also does the same thing by analysis; for when analysis leads thought up to the more simple and general elements, and also distinguishes both genera and differences according to the nature of each, then the assembling composition assembles the different elements and the simples together and marks off each part of the subject-matter of mathematics.

Thus mathematical science provides its own definitional account from itself and can discover it by inspection. However, since division makes the one many and definition the many one, in both directions mathematics must study the unitary one from which it begins and to which it is led back. So this will be the goal, which one might also call the form, which will be a unity covering all the investigations; this itself is both fine and good, and to it the divided parts strive to be attached. So for this reason we, as many of us as study mathematics in 66,1 accordance with Pythagorean precepts, always discard also in mathematics the unlimited and the particular, and rather endeavour to attain the general and the definite, until we bring up the whole study of the field of mathematics to a unity of all parts of mathematics together. This is the goal towards which those who truly wish for a view of the totality of the forms of mathematics must strive.

Chapter 21

Who were the originators of Pythagorean mathematics, and what according to him are the salient points of such a science; how, according to him, one should organize mathematics. A general survey.[139]

But since we are especially enquiring into Pythagorean mathematics, but it is not possible to give a complete account of it unless one comes to see its earliest

beginnings, for this reason it is necessary to include also those who were the predecessors of Pythagoras in such a study for the purposes of our present enquiry. For our examination of it would become as complete as possible if we were to establish it by beginning from its first roots.

Now they say that Thales[140] first discovered a good many things in geometry and passed them on to Pythagoras.[141] So we should rightly assimilate the mathematical investigations of Thales which we have received from him to Pythagorean mathematics. After Thales, Pythagoras spent a long time with the Egyptians[142] and he derived from them much of value for mathematical science. Therefore we should not be acting improperly in including alongside as well many things from Egyptian mathematics. But since he also later associated with the Assyrians and those among them who are called Chaldeans[143] (for this is what their mathematicians are called), it is necessary for us to accept much from them into our approach to mathematics.

Chapter 22

The distinctive practice of mathematical science according to Pythagoras, and how many uses of it he envisaged for the soul and for mankind; also how they practised it throughout the whole of their own life.

But this is not after all a sufficient account. Rather, since Pythagoras added much from his own store to the mathematics he had received from the barbarians, we must both include such beginnings and add also an account of the special character of his mathematics. For[144] he studied much of mathematics in a philosophical way and adapted it to his own focus (*epibolê*), even though it was handed down by others; also he imposed the proper order on it and made fitting lines of inquiry about it, and he always throughout provided coherence, so as nowhere to transgress the bounds of consequentiality.

So we should conform to these principles in following in the track of Pythagorean mathematics. But let us take select parts from it as being common elements, so that we may learn from them the symbolic and unfamiliar use of mathematical terms.[145] For since he was aiming at realities and truths he therefore applied their natural names[146] in mathematics. He made from them a

starting point of his teaching, capable of guiding his students if by sufficient experience they understood the relevant terms sufficiently. Indeed, by the clarity of his demonstrations, their elegance and accuracy, he excels all study by others in the same field; he employs much clarity, and he starts from what is familiar. But the finest aspect of his work is actually[147] the loftiness[148] of thought which leads up to the primary causes, which pursues its studies for the sake of the facts, which seizes purely on realities, and which sometimes conjoins mathematical insight with theological. One might put forward this much for the present[149] as select common elements of such a science.

Now as regards how one should engage in the pursuit of this science, it is worth making the following summary remarks, in conformity with what has been passed down by these men. So then, since the great bulk of this doctrine was actively present to their minds, and was therefore preserved unwritten in memories which now no longer remain, none of it can be easily vouched for or discovered either from writings or from hearsay. So something like this must be done. We must start from small clues and continually build them up and increase them; we must take them up to suitable starting points, interpolate what is missing, and speculate so far as possible as to their doctrinal positions, what they would say if it were possible for one of them to inform us. But as it is, from pursuing the logical consequences, we are able to uncover their teachings suitably from what they have indisputably handed down to us. For such methods of investigation will enable us either to arrive at the actual Pythagorean mathematical science or to come very close to it, to the highest possible degree. I have reached the conclusion that in this way I was agreeing with its practice as it was carried on according to its own originator. For it was in every way peculiar to him, and it stands out from other disciplines, since it pays attention to the soul and to the purification of the eye of the soul, provoking the discovery of the primary forms and causes of the being of mathematics[150] and harmonizing it with the nature of things themselves. It assimilates it to the intelligible forms and teaches its kinship with the Good and the community of the branches of mathematics with each other.

Since, then, the mathematical discipline was of this character, it searched out earnestly and keenly the theorems in its province without intermission. It contributed to the soul clarity of knowledge and subtlety in reasoning, accuracy of argument, contact with the incorporeal entities involved in it, well-

adjustment, harmony, and conversion to the world of being. Also for mankind it brings order into life, peace from passions, fineness of behaviour, and discovery of the other things that are of advantage to human life. They practised it throughout their own life, weaving the gain from it both into their activities and into the conduct of their souls, the organization of their cities and the management of their homes; also into skilled work and the equipment for war or peace, and altogether they introduced mathematics into every aspect of life, suitably to the objects of activity, profitably to the agents, and with great care for both of these and correspondingly for everything else.

One must then follow in these footsteps not simply by practising mathematics as such; for the mathematics now popular relies too much on perception and imagination, is a stranger to truth, and has unnaturally turned rather to the world of becoming. But if we should wish to study mathematics in a Pythagorean manner we should follow eagerly its divine route which raises, purifies and perfects.

Chapter 23

That the Pythagoreans extended mathematics not heedlessly, but with a view to the life of necessary utility;[151] *a record of the reasons for this, with many examples.*

70,1 Consequently[152] it is easy to comprehend also that it was not heedlessly that Pythagoras converted philosophy[153] concerning mathematical matters into a pattern of liberal education, and advanced it much by the wealth of what he revealed and the exactitude of his demonstrations, and studied it beyond its necessary use in everyday life. For if we have acquired some seed and starting point of such knowledge from which, having previously accepted verbally the kind of science it is, we have come to see exactly what its nature is, this has come about for us from nowhere else than from these.[154] But also, the potential of this science became plain through its own type of arguments being employed in the demonstrations in this field.[155] Further, when we were heedlessly placing our trust in the multiplicity of appearances,[156] the understanding of these [revelations] set us right by establishing how the truth stood about them. Most importantly, we first participated in the free-minded vision suitable to

philosophers by participation in these studies. For to each man is suited what is similar to him in nature, and for the free man the supreme goal of his activity with regard to his own life has reference to himself and to nothing else external
<– which indeed>[157] is the character of the contemplative sciences listed, and which characterizes them primarily, because their study holds the first place at the time of youth, and needs no such inductive process as naturally arises from familiarity with particular instances.

But if one may assign to this person some name suitable to the nature of his desire,[158] as one does in the case of other enterprises, which are named from a predilection for one sort of object or another, the philosopher seems to have a
drive towards some science that is to be prized for its own sake and not for the sake of some among its consequences. For some of those who wish to advance mathematics, but state that we should learn it because training in it is useful for other studies, for the sake of which they encourage us to do so, would not seem to the Pythagoreans to assign its proper place to it. For the things for the sake of which they advise us to act are by their nature, and also by the accounts customarily given of them, less akin to truth, nor are they in the same class for the exactitude of their demonstrations. Here is a sufficient sign of this: we see mathematical proofs lasting and being trusted throughout equally by all those who participate in them, but of those in other studies we should find very, very few that are such.

So mathematical philosophy has been a help to us already for many things necessary for living and for optional extras[159] and also for things that are valuable for their own sake. For we should find help being received from mathematics by not a few manufacturing crafts; and also in the case of natural philosophy, and indeed any other area of philosophy holding a more esteemed rank than that, we shall find it using in its own demonstrations many of the things we have discovered by the methods described. Further, as mathematics makes us akin to the orderly and to order, it should provide some impulsion towards excellence and to everything fine.

But one would not cherish its power only on account of such help, but, still
more, for itself and its own nature. For it is agreed that some sciences are choiceworthy for their own sake and not only for the results they bring about. Possibly this is so for the theoretical sciences alone, since they have no other objective beyond research (*theoria*). We judge one science to be more valuable than another by the same standards as we judge each one to be valuable. We

value one above another either because of its exactitude[160] or because it studies superior and more valuable objects.[161] Of these, everyone would agree with us that the former is especially true of the mathematical sciences, and those would agree to the latter who assign the pre-eminence mentioned to primary principles and believe that numbers, lines and their features properly possess the nature of a principle because of the simplicity of their being. Again, theorems about the heavens, which have the most honoured and divine rank among things naturally perceptible to us are recognized by astronomical[162] science, which counts as one among the mathematical sciences. But it would seem to be absurd and by no means generally agreed that, while professing that the philosopher possesses a natural affinity with truth, one should think that he should look for some other profit from such theorems which have shared in the highest truth; and that we should expect that he should take to them for some other reason while engrossed in such sciences as are concerned with the most general facts about nature and the most divine of those concerned with what is perceptible to us – sciences which are full of the most abundant and wonderful insights, and have an exactitude not fabricated from empty words but appropriate and well-founded[163] through their fundamental nature.

In general, of those characteristics that one would expect to be present in sciences worthy of choice for their own sake, we shall find the mathematical sciences to share in all of them. For each of them is concerned with some nature, and one that is eternal, and that contains in itself insights many and wonderful, both through the order of their own features and through their abstraction from conclusions derived from experience. Further, in taking as premises of their demonstrations ones that are well known and self-evident, they construct arguments based on them in these matters in such a way that the demonstrations in this field are a model for those who would wish to draw accurate conclusions.

Therefore mathematics would seem to fit in well for those who think that a course of life spent in philosophizing is worthy of choice for its own sake, and that mathematical study is related to and akin to philosophy.[164] So, as one would expect, for all these reasons the Pythagoreans thought highly of zeal for mathematics, and they coupled it in many ways with the study of the cosmos. For example, they included number in their calculation from the (heavenly) circuits and their differences; they viewed what was possible and what impossible in the constitution of the cosmos from the possibilities

and impossibilities in mathematics; they understood the heavenly circuits with good cause through commensurate[165] numbers; they determined the measurements of the heavens by certain mathematical ratios; and in general they established a science of prediction in nature based on mathematics, and, in respect of all the other theorems proper to the cosmos, they made use of mathematics as principles. From these they both contributed many demonstrations to the study of nature, they direct our virtuous impulses towards the fine and the good, and, most importantly, they pursue astronomy[166] at the level of theology through mathematics. So, for all these reasons, it is natural that they should exhibit a remarkable zeal for this discipline.

Chapter 24

What was the group practice in the Pythagoreans' occupation with mathematical pursuits and what was their training and preparation in sciences?

Next, we should discuss the group practice (*sunêtheia*) in the Pythagoreans' occupation with mathematical pursuits. So then, they separated mathematical arguments from the realm of the sensible, and converted the reasoning faculty[167] to an assurance of incorporeal being through mathematics. They used it as a means of transport to the intelligible, and especially investigated what mathematics contains akin to the pure forms and unitary reason-principles. Therefore they made use of this procedure in their inquiries, once and for all separating the knowledge of mathematics from the common and widespread level of understanding, and accordingly transmitted it as a secret. They shared the knowledge of it with very few, and if some leak to the multitude occurred, they abominated it as an impiety. Consequently they repelled also those outside the group practice as being unworthy to share in mathematics. For Pythagoras[168] considered that they should not share their mathematical knowledge with all, but only with those with whom one would share the whole of one's life. He did not accept people into this fellowship at random nor casually, but he tested them over a long period, rejecting the unworthy.

Also, he did not communicate outside the group practice the advances made by himself, but kept the discussions of mathematics secret from others; but he

brought about great advances among those called Pythagoreans by their association with him, both in the understanding of mathematics and in the study of geometry; also one would find that the basis of nearly all later advances came to us from him. He esteemed in mathematics not, as do some later persons, the power by which they will be able to solve a problem, but the insights themselves; and, of these, not those most difficult to discover, like most of those later, but those of them in which one could recognize order or some natural property.

This was their attitude because they thought that the principles of the whole of nature were present in these, and that these were especially easy to grasp – what they were and how many – because they were concerned with a nature that was invariant and free from change, and was also simple. Therefore they did not deal with issues that involved raising problems except those concerning basic elements, such as geometrical application (*parabolé*) and the squaring <of the circle>.[169] Nor did they make it their business in their investigations to exhaust every issue and leave no possibility unexplored, but sought only to see the principles themselves in each case. They made a training in these sciences and a rational working out that was exact and theoretical into a proper science. They assigned a suitable ordering within the sciences, and they accepted few things as principles and developed them. Also they especially perfected the most valuable and most profound of the theorems; they applied theorems differently studied to other matters, and set them in such an order as to propound the more simple <first>, those requiring composition next. Also they arranged them in accordance with the nature of things, suitably to our power and to the worth of those who received them,[170] in a way leading to excellence, in accord with the whole of education and as proper for the purification of the soul.

Such, then, is the information one might give about this Pythagorean method of procedure, about which we shall say more in the accounts that will be given of each of the branches of mathematics individually.[171]

Chapter 25

Which of the Pythagoreans were Mathematicals, and in what respect did they differ from the Acousmatics?[172] *What was their function and of what kind the form of their arguments and demonstrations?*[173]

There are two types of the Italian philosophy[174] which is called Pythagorean; for there were also two classes of those who took part in it, the Listeners and the Mathematicals. Of these, the Listeners were agreed to be Pythagoreans by the others,[175] but these denied that the Mathematicals were, or that their method of procedure derived from Pythagoras, but rather from Hippasus. Some say that Hippasus was from Croton,[176] others from Metapontum. Those of the Pythagoreans concerned with mathematics admit that the others are Pythagoreans, but say that they themselves are still more so, and that what they themselves say is true.

They say that the cause of this difference was as follows: Pythagoras arrived from Ionia, and specifically from Samos, at the time of the tyranny of Polycrates[177] and when Italy was flourishing, and, they say that the leading men in the cities became associated with him. With those of them who were older and busy, since they were involved with the affairs of the city, he conversed non-technically, because it was difficult to talk with them in mathematical terms and through formal proofs. For he believed that they would be no less helped without knowing the explanation of how one should act, just as patients no less achieve health without also being told why they should do each thing. But with those with whom he conversed who were younger and able to work hard and learn, with such he conversed through the medium of demonstrations and mathematics. They themselves, they said, were descended from these, the Listeners from the others. With regard to Hippasus they say that he was indeed a member of the Pythagorean community, but because he first published and wrote up the twelve-sided figure composed of twelve pentagons,[178] he perished at sea by reason of having committed an impiety and assumed a reputation as the discoverer of the theorem, whereas everything was the work of 'that man'. For that is how they refer to Pythagoras, and do not call him by his name.

Mathematics flourished, since two pre-eminent mathematicians, Theodorus from Cyrene[179] and Hippocrates from Chios,[180] developed it. The Pythagoreans say that geometry was developed as follows: one of the Pythagoreans lost his money, and since this misfortune had befallen him, it was granted to him to make money out of geometry. Geometry was called 'inquiry' (*historia*) by Pythagoras.[181]

So then, with regard to the difference between each mode of procedure[182] and with regard to mathematics, what happened was more or less as described

above. The Pythagoreans spent their time on mathematics and espoused the exactitude of its arguments because it alone among human undertakings contained demonstrations, and, seeing that the sciences of harmony in terms of numbers and of optics through diagrams were equally universally accepted, they considered these to be in general the causes of existents and their principles. So, they thought, anyone who wishes to view the nature of existents must look to these, numbers, the geometrical forms of things, and definitions, since all things were made clear by these. So, not having attached the powers of each of themselves to anything more timely or valuable than to the causes of all things,[183] they also determined other matters as well by these in much the same way. So the training in numbers and mathematical questions seemed to them to be through these and by this method.

Such then[184] was the style of demonstration that they used, proceeding from such principles and thus exhibiting trustworthiness and security in its arguments.

Chapter 26

Objections to mathematics as being worthless; replies thereto, and comparisons of these arguments in detail.

79,1 There[185] have been some,[186] however, some ancient, some modern, who have advanced the opposite opinion about mathematical sciences, condemning them as being utterly useless and contributing nothing to human life. Some frame their attack as follows:[187] if the goal for the sake of which the philosophers say one should study mathematics is useless, there is a much prior necessity that the study of it itself is pointless. But concerning its goal, practically everyone who has studied it most accurately would seem more or less to agree. For some say it is knowledge of injustice and justice, of goods and evils, which is similar to geometry and other such studies, others that it is wisdom about nature and such truths as are related to this, of the sort that the followers of Anaxagoras and Parmenides proposed. But it should not escape him who intends to research on these matters that all goods and all things useful for life are such by virtue of use and in action, and not by virtue of mere knowledge

about them. For we are not healthy by knowing what things are productive of health, but in applying them to the body, nor are we wealthy by acquaintance with wealth, but by the acquisition of much substance. Nor, above all, do we live well by the mere knowledge of any things, but rather by good actions; for in truth that is what happiness (*eudaimonein*) consists in.[188] So it is fitting for philosophy also, if it is to be of use, either to be the practice of good things or to be instrumental towards such practices.

Now it is clear to all[189] that neither [mathematical philosophy] itself nor any one of the aforementioned sciences is a making of things. From that one may gather that it is also not useful for actions. For[190] we have as the best illustration sciences similar to it and the beliefs underlying them. For of the things which the geometers study by means of demonstration, in none do we see them competent to engage in practice. Rather, the division of an area and all the other operations on magnitudes and places are rather within the competence of surveyors by reason of their practical experience; while those concerned with mathematics and the accounts of these matters know how one ought to do it, but cannot do it.

It is the same with regard to music and the other sciences in which the fields of knowledge and experience are separate. For those who provided the demonstrations and syllogisms concerning concord and similar matters, just like those engaged in philosophy,[191] are accustomed to being spectators, but they do not share in any of the relevant activities. Rather, even if there happen to be some of them who can use their hands, when they learn the demonstrations, just as one would expect, they straightaway perform worse. But those who are ignorant of the theory but are trained and have a right belief are in every way superior in practice.

It is the same with regard to those concerned with astronomy,[192] for example about the sun and the moon and the other stars; those who have studied carefully the causes and explanations know nothing of what is useful to people, but those who possess what they call nautical sciences can predict for us storms and winds and many other happenings. So such sciences will be utterly useless for practical activities; indeed, if they fall short in correct practical activities, then scholarly research fails to achieve the greatest goods.

In reply to this, we say[193] that mathematical sciences exist, and that these are capable of being participated in. For both things prior and things better in

nature are always more intelligible than the posterior and the inferior. For they are knowledge of matters that are definite and orderly rather than of their opposites, and also of causes rather than of consequences. But[194] what is in the field of the unchanging forms of mathematics is definite and orderly. Also things prior are causes rather than consequences; for, if they be taken away, so are things which derive their being from them, such as lengths from numbers, surfaces from lengths, and solids from surfaces. So since the entities of mathematics are more simple than all others they will be most fundamental of all. So sciences concerned with matters superior and more fundamental will be much more sciences, and capable of being acquired; for it is a much prior necessity to have knowledge of causes and elements than of things posterior to these. For those are not concerned with basic issues, nor do the prime elements come from them; but it is from these and through these that the rest also come to be and manifestly are composed.

It is clear from these considerations that knowledge of mathematics is the greatest of goods and more useful than all others. For reason and wisdom are foremost among goods. There is, after all, no one except the wise man to be a standard and most accurate determinant of goods; for whatever he would choose, those things are good and the opposite of these are bad. But since all men mainly choose what is in accord with their own dispositions (the righteous man a righteous life, the brave man a life of valour, the temperate temperance), similarly it is clear that the wise man will choose wisdom above all things; for that is the function of that faculty. So it is plain that according to the most authoritative judgement wisdom is supreme among goods. Also,[195] we should not pursue wisdom at all for utilitarian purposes, for it is actually choiceworthy in itself.

The usefulness and magnitude[196] of this discipline have now, I think, been sufficiently demonstrated; it is the following considerations that convince me that its acquisition is much easier than that of other goods. For the fact that philosophers, without getting payment from people, for which they would accordingly work strenuously, and despite having surrendered a good head start to the other disciplines, have nevertheless, though running a short time, overtaken them[197] in accuracy, seems to me to be a sign of the easiness of philosophy. Also the fact that all feel at home with it and wish to have leisure freed from all other matters is no slight evidence that application to it is

pleasurable; for nobody welcomes a long period of drudgery. In addition, its practice is exceedingly different from that of all the rest. For it has no need of tools or places for its pursuit; for, wherever in the world one locates the 83,1
reasoning faculty, it grasps truth equally everywhere as if it were present. But[198] this is perhaps quite enough to say for present purposes; for it has been demonstrated that wisdom is possible and why it is the greatest of goods and 5
easy to gain possession of.

Now accurate argument about truth is incontrovertibly the most recent of practices. For after the destruction and the flood[199] people were at first compelled to devote their thought[200] to subsistence and keeping alive. When they became better off they invented the skills that aim at pleasure, such as 10
music and similar skills; but when they became well provided with necessities they accordingly set themselves to practice philosophy. But now the geometers, and those concerned with argument and other education, have progressed in 15
their enquiries so far from small beginnings and in so short a time[201] as has no other type of enquiry in any of the arts. However, all join in promoting other skills, honouring them by general consensus and giving rewards to those who possess them; but we not only do not encourage those who are occupied with the mathematical sciences, but in many cases actually put obstacles in their 20
way. Yet nonetheless these progress most of all because they are by their nature the most fundamental; for[202] what is later in its establishment is prior in its essence and perfection.

So, then, mathematical science is supreme over the other sciences in all these respects, being easily foremost in beauty and in exactness among all 25
pursuits.[203] This is reasonable in the following way: first of all, things related to 84,1
the realm of generation are most sought after by men, with the aim of possessing them as far as possible; but after these, those things that free us from our bodily nature are much more valuable than the former. For the former confront us as 5
necessities, but these latter are thought worthy of priority and honour as being choiceworthy in themselves and august.

So[204] mathematics also contributes not a few aids to the whole of human life, as is quite obvious to those who survey the contributions to life of mathematical 10
skills. And yet these contributions are worthy of less consideration; most important are the purification of the immortal soul, the turning of the intellect to the intelligible world, and participation in the activity of the world of being.

In providing us with these benefits, mathematical science produces all things good, so that I know of no other approach which so draws us to the goal of happiness.

In these ways, then, not only have opposing accounts been revealed as false, but also mathematics has been demonstrated to be of the utmost value to us.

Chapter 27

What the really educated person should ask for from the mathematician; also how one should judge his study and by what standards determine its correctness.

Now,[205] since the function of an educated man is to be able to judge successfully what a speaker contributes for well or ill, we believe the well-educated man to be such a person, and that being educated is the capacity to act in the way stated. It is indeed clear that the rightly educated man should, in the case of
85, 1 mathematics also, require from the mathematician correctness and his special contribution, whether he well or badly conducts the study of mathematics. For just as we consider that the fully educated man is more or less a judge of all matters,[206] as a single individual, so with regard to any specialized science there should be another who is placed similarly to the above-mentioned with regard to the corresponding section. So it is clear that with regard to mathematics there should be some similar standards by reference to which the educated man will approve the manner of the demonstrations, quite apart from the question of what is true, whether thus or otherwise. I mean, for example, whether we should take each individual theorem of the mathematicians and make a decision about it on its own – e.g. about these particular triangles – or whether we should examine the general theorems and what holds of all, with common assumptions. For many identical characteristics belong to many kinds that are different from each other; for example, one might construct a proof in so far as something is a triangle or in so far as it is, generally, a rectilinear figure. For if certain identical things belong to different species, there need be no difference in their demonstrations. Things may perhaps be different which happen to belong to the same category while differing in species; for example, similarity in the case of triangles is one thing, in the case

of numbers another. Also, one must in each case construct specific demonstrations. So one should consider when the theorem should be generic and when specific; for to distinguish between these makes a significant contribution to mathematical education.

Further, one should demand of the mathematician accounts suited to the underlying substance,[207] and that he should carry out the type of demonstration suitable to it. For just as we accept plausible arguments from the orator, so one must demand necessary demonstrations from the mathematician.[208] One should not, after all, seek the same degrees of necessity everywhere, nor, similarly, the same accuracy in all matters, but just as with regard to crafts we divide them by their underlying matter, not requiring the same accuracy in gold, tin, and bronze, nor in cork, boxwood, and nettle-tree,[209] it is the same in the theoretical sciences as well. For the subject-matters will impose differences, when some are simpler, some more composite, and some altogether unchanging, others subject to change,[210] as for example the objects of arithmetic and of harmony, or those of geometry and of astronomy.[211] Also, in some of these intellect is the principle, in some discursive reason, and in the case of yet others there may be some small stimuli arising from sense-perception, as in the case of the heavenly bodies. For it is not possible to bring to bear the same or similar explanations regarding such things, but to the extent that the starting points are different, so must the demonstrations be different; for in each case the manner of proof is cognate to the subject-matter.

A still greater difference than these is that in investigations some proceed, and some do not, from first principles; so that not even here can one specify that explanations or arguments are similar in kind. In this situation it is necessary to recognize what they have in common and what is different, and what is analogously the same, and what sorts of topics require more data, and in which of them the problems are more numerous; these, then, are the differences in these and similar matters between the demonstrations and arguments in each. Such an awareness would contribute not only to correct judgement but also to correct research, for by possessing a distinction between what counts as explanation in each case it will make the arguments germane, which is not easy for one not accustomed to this. For nature itself on its own can lead us to fundamentals, but does not suffice to judge each case without the assistance of a different level of understanding.

Also, one must distinguish whether there are many causal principles about which the mathematician must speak, and which of these are naturally prior and which secondary. For he who is educated mathematically will be capable of examining also the explanations provided, and will be adept at studying their order.

This also must not escape our notice, that many of the more recent Pythagoreans were of the opinion that the only subject-matters of mathematics were the constant and unchanging, and they hypothesized only these as principles; and so also in the same way they set apart the sciences and demonstrations concerning such things. But since we in the present discussions and those which are to follow later will demonstrate that there are many other entities which are constant and unchanging, not only those of mathematics, and that these are superior to and more to be valued than the mathematical, we shall also demonstrate that these mathematical principles are not the only ones, but that there are also others, and that these are superior to and more
88,1 powerful than they, and that the mathematical are not the principles of all that is, but of certain things.[212] For these reasons mathematical demonstration requires a distinction of what sort of things constant and unchanging it is demonstrative, and from what sorts of principles it reasons, and about what sorts of problems it provides proofs. For the education which is a judge of such matters defines the correct procedure and the goal of mathematics, and it makes this judgement of mathematics as is needed, and it embraces admirably the way in which research also should be conducted. So[213] let that be our specification of these matters.

Chapter 28

When the problem or the method of demonstration is mathematical or belongs to another science; a scientific distinction.

Since[214] it is often not clear whether a projected piece of research is mathematical or within the ambit of some other science, and the method of demonstrations is debated as to which is mathematical, it is necessary to examine these matters carefully as well, to discern what sort of problems and what method of

demonstration reveal a mathematical approach. In general, then, one must know in advance how theological and natural science are related to mathematical study, and how the demonstrations and problems of these sciences are connected with each other. Now[215] the kinship of knowledge regarding the differing methods of learning is truly the case, and their subject-matters, which are in a way akin, provide the shared continuity of the sciences with each other.

However,[216] the Pythagoreans still more closely connect up this indivisible association, since they hand down much about the intelligible forms mathematically, and they teach mathematically much about nature and much also about ethics. However,[217] one must distinguish these three kinds of account and define the mathematical one distinctively, so that it is not confused with any other of them.

So let[218] mathematical lines of argument (*logoi*) be unchanging as such and irrefutable, fitting to mathematical species and genera as such, receiving them not by abstraction from perceptible things, but focusing on them separately as they exist as such, free from change, but not merging into identity with the intelligible and indivisible forms, nor with intellections, but proceeding in accordance with things intelligible and connecting up the reasoning about mathematical objects with these, and participating in knowledge in a sort of intermediate manner. For such an approach as this to arguments and demonstrations is mathematical and far removed from the accounts given by the other sciences.

Chapter 29

Concerning the mathematical syllogisms, and the mathematical divisions and definitions; how mathematical science should use them, whether it gets its principles in its own way or from dialectic.

Following on such problems, we may next examine such a question as the following: how mathematical science makes use of division, definition, and syllogism, whether it receives understanding of these from dialectic or initiates its own activities in these matters. If it does take over these from the study of arguments, it will be deficient in many respects and will be deriving

from elsewhere a share in the principles of its own knowledge. But if that sort of situation fits some other rather than mathematical exactitude, one must grasp the fact that these three, i.e. division, definition, and syllogism, which have one character in the context of dialectic, another in that of mathematics, are to be distinguished from each other in accordance with the special character of each. Thus the rules of dialectic are wider, and a thorough account of them is not now required; but those of mathematics, which are special to it, are uniquely distinct to mathematics. So it discovers them out of its own resources, perfects them and formulates them, and it has no need of another science for its own field of study. For it does not examine things without restriction, as does dialectic, but rather things that fall under it, and it studies them in its own way in so far as they fall under it. About these it produces exact definitions, and it also possesses within itself criteria[219] by which one should approve the definitions for oneself.[220] Also, it constructs many kinds of demonstration, and among these it distinguishes the better and the true from those that are doubtful and in need of closer examination.

It is actually engaged in a double activity, one sort contributing to discovery, the other to critical analysis. It is critical and inventive in accordance with the special rationale of its own skill, not with that of unrestricted theoretical intellect. So it is able to determine by this rationale how one should distinguish species in mathematics, what the appropriate differentiae are that differentiate them, what definitions in mathematics are, and how they should be assembled from mathematical divisions; how a mathematical syllogism comes about and through how many distinctions it achieves exactitude; when it is capable, when incapable, and in how many ways it exhibits necessity. For in all these matters its own proper method of discovery and use and judgement has no need of imported apparatus, as some believe that it has.

Chapter 30

That mathematics makes great contributions to all of philosophy and to all its parts, helping it in every respect; and especially that of the Pythagoreans, which is very different from the other mathematics.

That mathematics makes many great contributions both to all of philosophy and to all its parts and also provides help towards the vision of realities, joining in the hunt for it, is easy to see from the above.[221] The mathematics of the Pythagoreans is not such as others practice it; for that is for the most part applied, without a sole target, and not aiming at the fine and the good. But that of the Pythagoreans is especially pure (*theôrêtikê*) and relates its insights to a single goal; it relates all its own arguments to the Fine and the Good, and uses them as means of access to the realm of true being. Setting out from such a starting point, it makes a fine distinction between its contents, as regards which of its studies are well fitted to theology and are able to share in the ordering and measures of the divine, and these it assigns to such a part of philosophy; and which, on the other hand, are suited to the pursuit of true being and to familiarization and harmony with it and conversion to it, and such studies it assigns to the part of philosophy proper to that. Nor does it fail to see that some studies are scientifically engaged with exactitude of argument and serve as guides to excellence in reasoning, demonstration, and definition, refuting the false and distinguishing the true from the false. Nor is it unaware of the appropriate harmonizing with the science of nature, how it is constituted, how it is useful, how it supplies things lacking in nature, and how it distinguishes among them. 91,1

However, it also descends to the construction of political constitutions and ethical questions, and reveals the correct form of life and the appropriate standards for household and cities through mathematics, and uses it as needed for the sake of excellence and for moral improvement and the best education, and for due measure, protection from evils, and the obtaining of what is fine, being thus suitably connected in common through all branches of mathematics and through each of them. Also, indeed, with regard to natural goods and the beneficial products of the crafts it has achieved the discovery of some things, while identifying others as accessories, and designating them as supplementary. It has provided products from itself and contributed parts to some others, and has thus perfected human life, so that it is self-sufficient and lacks nothing that is needed for living. So then,[222] we might reasonably coordinate the type of mathematics practised by the Pythagoreans to philosophy, as being akin to it and befitting it. 92,1

Chapter 31

That the Pythagoreans used the same mathematics for many different purposes; that they conducted many studies that clarified the same problem, and for what reasons they did that.

Since Pythagorean mathematics is of this character, we find it making use of its knowledge of matters in many directions. For[223] it uses the same mathematics for many purposes, whether natural or theological, or as concerned with coming to be or the elements, or issues related to generation, whether complex or simple. For example, it uses number in all these connections, as well as harmonics and figures and other similar devices. Also sometimes it makes use of many of them simultaneously to throw light on the same subject, as, for example, in the case of the soul[224] it makes use of number, harmonics and figures and other mathematical entities. The explanation of the former of these practices is that the nature of each branch of mathematics and each mathematical entity is complex in form; for that is why we use the same for many purposes – for example numbers for theological and natural questions. For the second practice the explanation is that some things are compounded from many substances and are intermediates between a number of different extremes. For that is why the soul and all intermediate natures are treated of in many branches of mathematics, as being able to fit together with many mathematical entities, and as receiving their settled character by derivation from many sources. The above, then,[225] may serve as an explanation of these matters.

Chapter 32

How sometimes we tackle also questions about perceptible things mathematically, and how often this occurs; also how in mathematics many things lead on to others and why this is so.

Mathematical investigation is accustomed also to dealing on occasion mathematically with problems about perceptible objects,[226] as, for example, to dealing with the four elements[227] by means of geometry or arithmetic or harmonics, and similarly with other matters. For since pure mathematics is

prior in nature and is based on things prior to natural objects, for that reason it also constructs its arguments as being demonstrations from prior causes. It does this in many ways: either by abstraction[228] (*aphaeresis*), as when it abstracts enmattered forms from their matter and surveys them mathematically; or by juxtaposition (*epharmogê*), as when it applies mathematical reason-principles to natural objects and fits them against each other; or by completion (*teleiôsis*), as when it completes bodily forms that are incomplete by adding what is missing; or by assimilation (*apeikasia*), as when it regards things in the realm of becoming that are equal and commensurate in the way in which they most resemble mathematical forms; or by participation (*metokhé*), as when we inspect the way in which the reason-principles of things in other areas participate in pure reason-principles; or by reflection (*emphasis*), as when we view the obscure trace of the mathematical mirrored in the sensible; or by division (*diairesis*), as when we recognize the single and indivisible mathematical form as shared out among the particular and multiplied; or by comparison (*parabolé*), as when we survey together the pure forms of mathematics and those in perceptible things; or by explanation from things prior, when, having first established the mathematical explanatory principles we survey them jointly to determine how the perceptible instances are derived from them. For it is in these ways, I think, that we undertake all matters natural and in the world of becoming on the basis of mathematics.

It is because of this, indeed, that many parts of mathematics do not remain at the level of mathematics, but are drawn into inferior areas, either through the choice of those who use them or also because of the kinship of perceptible facts to them. But sometimes, what begins as mathematical is led up to greater things,[229] and into all other matters, because its incorporeal essence is well-suited to being treated as akin to the pure essences of beings, and also because mathematics is of a nature to be assimilated to all things.

So much, then,[230] about these matters also.

Chapter 33

What is common to the whole of mathematical science and specific to it with regard to the differences observed in its many species, and how one should cut it

in accordance with scientific division from one into two and then into more species.

Since the common genus of the whole of mathematical science is of central importance to understanding the present study, it is especially necessary to see clearly in what respect the whole of mathematics has a common identity. For if we see clearly this[231] single genus that it has and thus grasp its common being we shall have the most complete understanding of it. Now we have already also at the beginning given preliminary consideration to this,[232] but we must also at the end put a crown on it in the same way as in our introductory preliminaries. So now let us once again undertake the account of the single genus of mathematical study.

So I declare[233] that it is to be precisely the intermediary between the intelligible and the perceptible classes of being that is the common genus of this science, which embraces within itself all the different species whatsoever, primarily participating in the kinds of things that have real being, but containing within itself also the kinds of perceptible things, while in every way superior to them in clarity, exactitude, delicacy, and incorporeality, possessing all kinds of powers, some leading up to the realm of true being, others declining towards becoming, and possessing levels of knowledge correspondingly.

Having established, then, a unity of this kind, let us take notice of the differences in it by reference to the dichotomies[234] of this intermediate nature. As one such dichotomy let us distinguish between the part connected with real being and the other which lays hold of perceptible things and stands prior to them. Again, with respect to each of these species, we shall grasp, by adopting ever different divisions, the intermediate position of mathematical objects in different ways, making distinctions either quantitatively or in some other way. And the distance from real being, whether to a further or a lesser degree, will provide the differentiation within mathematical objects. Further, the order of priority of the things comprised in the universe, whether primary or secondary, will itself also bring about many differences in mathematical objects. It behoves us to discover their order, either by succession, or by continuity, or by contiguity, or by some other form of connection.

We make these distinctions by virtue of this structuring that proceeds and multiplies itself from unity and in turn by the grouping together which leads

up to unity, and, while so distinguishing, we in the same manner connect it all together. Also in this way the common essence of mathematical objects will be encompassed generally by calculation and analysed, and it will be raised again from its multiplicity to the single genus of the essence of mathematics. And this, after all, is the goal of mathematical science in its role of dividing and distinguishing.

Chapter 34

Whence mathematical science got its name and what its characteristic is; also to what one should pay attention in judging the species of mathematics.

The science of mathematics is so named since a certain acquisition of knowledge (*mathêsis*) descends to it from the intellection of things intelligible, and this study is akin to that intellection. But the objects of intellection and understanding are also objects of which one can acquire knowledge; about these things alone among others can knowledge be acquired, which is why they are called 'acquired knowledge' (*mathêmata*).[235] It seemed to the Pythagoreans that these mathematical sciences were provided as being appropriate tools for the revelation of the nature of real beings, and that it removed all the fog[236] which obscures our view of reality, so that the truth itself should be seen.

Furthermore, it seemed to them that such an acquisition of knowledge would also provide the explanation of ethical consensus, that it would at the same time reveal the nature of good and evil, becoming an aid to philosophy, and that by it the order within the universe and symmetry of the cyclical motion of the heavens would be revealed. Therefore they thought it impossible to philosophize without it. Also it hunts out, as if by means of mirrors outstanding in clarity, many representations[237] of the workings of nature, which is what they called the mathematical part of philosophy, and they called those experienced in such accounts mathematicians. They also thought that the objects of mathematics were the best representations of the contents of this realm, because these were appropriate for intellection, being enduring, unchanging, and for ever remaining the same, by turning one's gaze onto and

imitating which one might produce results that were in all cases stable and firmly based.

Mathematical reasoning, then, possesses clarity and a scientific and irrefutable character. Indeed, the mathematical seemed to the ancients to be superior to and more fundamental than that of natural philosophy; for they took it that from it depended the intellectual apprehension of other things. That then is the character of mathematical reasoning. In judging its formal character, one must pay due attention to its exactitude, its irrefutable understanding, the propriety of its arguments, and its concord with the world of being; for it is in conjunction with these that its character is best displayed.

So[238] let the above be added as well to what we have previously expounded on these matters.

Chapter 35

Summary of the general account of all mathematics, an indication of the order[239] *of the chapters and also a reminder at the same time about the right way to divide up the whole conspectus of the discipline.*

Since, then, we have completed the general study of mathematics, the time has now come to draw together within a single conspectus the main headings under which it falls.[240] The first, then, are those concerning its targets and those concerning the whole of the mathematical sciences;[241] next those concerning principles, both the general and the specific.[242] After these we gave
98,1 an account of the subject-matter of mathematics,[243] and then of the best use of mathematics and the sciences it includes,[244] and about their internal criteria[245] and concerning in how many ways its determinate being is viewed;[246] also what the functions of mathematical science are,[247] and what its powers and elements, both those common and those specific,[248] and how all these make a joint contribution to philosophy.[249] Also in how many ways mathematics contributes to technology,[250] and what place it has in the process of education.[251] Also what are the special features of the Pythagorean tradition,[252] and what is according to the Pythagoreans the division of mathematics as a whole.[253] Also what the distinguishing feature of mathematics is[254] and what the most remarkable features of this study are according to Pythagoras,[255] and what

according to him makes its practice special.[256] Also that the Pythagoreans did not develop mathematics to its greatest extent accidentally,[257] and what their manner of pursuing mathematical studies was.[258] Also, who among them were the 'mathematicians',[259] the objections to mathematics and the replies to them and comparisons between them,[260] and what the truly educated man should ask of the mathematician.[261] Also, a distinction between the problems in it[262] and the character of the demonstrations,[263] and about mathematical syllogisms, divisions, and definitions in mathematics.[264] Also, the connection of philosophy with the Pythagorean method which the Pythagoreans used in mathematics.[265] Also, what is general and what specific in mathematical science and how many divisions it contains, and how they are ordered. And finally, something is said about the name 'mathematics' and other names consequent on it.[266]

These, then, are the main headings of what we have examined concerning the general account of mathematics and the mathematical sciences, and I think that they are in due proportion; but if some matters have been omitted, these also will receive an easy solution from what has been said. 99,1

Notes

1 What follows are chapter headings, which are repeated at the head of each chapter. RRKS. The thirty-five chapters were designed and provided with headings by Iamblichus himself. The Pythagorean sequence of Iamblichus was apparently the first time an author composed his own chapter divisions and headings in an ancient book of philosophy. The idea of a sourcebook constructed in this way represents one of his most original innovations in philosophical literature. The headings provided by Iamblichus survive in the manuscripts in two locations, at the beginning of the *DCMS*, and near the beginning of each chapter (in the margin); and these headings clearly go back to Iamblichus himself, according to internal evidence and also the witness of Syrianus (head of the Athenian school from 432–437 CE), who reiterates the chapter titles of *DCMS* 2–32 in his commentary on Aristotle's *Metaphysics* (101,26–102,35), translated by John Dillon and Dominic O'Meara in this series. From DSH/MRJ

2 'Essential being' smuggles in a view which appealed to Plato's Academy and later to Iamblichus, but which I do not find prominent in the early Pythagoreans before Plato. On the Platonist view, there are higher realms of reality, each with its own essence, the realm of Platonic Forms, above the realm of objects studied by mathematical sciences. RRKS

3 To me this chapter appears to be largely Iamblichus' own overview of the book, as announced in the chapter heading, which is repeated here to facilitate comparison with Iamblichus' wording in his overview. Nonetheless, even though not directly quoting here as he is through much of the book, Iamblichus is drawing on earlier sources, and several references to Plato are identified in the notes to the chapter. Many quoted sources are identified in notes to other chapters, but others still need identification. RRKS

4 *skopos* is a target that you *look* at in order to aim, and the term had a special use in commentaries on Plato and Aristotle where each work of theirs was thought of as having a single target, which was also, in English terms, the work's 'scope'. The target was thus a subject to be covered. But here Iamblichus uses a different word for the overall *aim* (*prothesis*) of the book and envisages a plurality of targets (to keep the metaphor intact) under that aim. RRKS

5 'Essential being': on *ousia*, see the note on 51,27. 'Essential being' here smuggles in a view which appealed to Plato's Academy and later to Neopythagoreans whom

Iamblichus takes to be earlier, but I do not find it in the early Pythagoreans before Plato. On the Platonist view, there is a higher realm of reality, the realm of Platonic Forms, above the realm of objects studied by mathematical sciences. RRKS

6 Chapter 2 below. RRKS

7 Chapter 3 below. RRKS

8 Chapter 4 below. RRKS. Iamblichus is here using as a source the injunctions of Socrates at Plato, *Philebus* 16D as to the correct manner of dividing up a subject for discussion.

9 9,13: *epibolê*. RRKS

10 Chapter 5 below. RRKS

11 Iamblichus' method of relying on quotation of sources, reserving the sometimes sketchy editorial comment for himself, fits with the literal meaning of *paradeixis* and *paradeiknumi* which might be rendered as 'indicating'. RRKS

12 Chapter 6 below. RRKS

13 *periagein*, *periagôgê*: a favourite word of Plato for total reorientation. See the note on 74,11. RRKS

14 Faculty of step-by-step reasoning, contrasted with *nous*, *noêsis* (intellect, intellection), associated with steady insight, that may, however, start with a sudden realization. See the note on 32,13–16. The divided line analogy of ch. 8 distinguishes four cognitive states, these two included. RRKS

15 Adopting the distinction made at Plato, *Timaeus* 35A.

16 The correct rendering of both *eidê* and *logoi* here is problematic, but Iamblichus seems to be making a distinction between mathematical entities as transcendent Forms and as 'projections' of those Forms in bodies. Since, however, below, at 11,13, *logoi* does seem to have to mean something like 'reasonings', it seemed best to give the same translation in both places.

17 This use of '*sunthesis*' for composition as opposed to division is different from the use at 59,8 (where see the note) for geometrical synthesis as opposed to analysis, although an analogy is noted at 65,15–16 between the two contexts in which synthesis is carried out. RRKS

18 The verb *diaporthmeuein*, '*transport*': cf. ch. 13, 50,24; ch. 24, 74,12. RRKS. It is borrowed from Plato, *Symposium* 202E, and seems to assimilate mathematical entities to a creative role in the universe, which is developed in what follows.

19 At this point a scholium or marginal note reads: 'The five kinds of being: 1. substance, 2. identity, 3. difference, 4. change, 5. rest – that is to say, the five 'greatest kinds' of the *Sophist*'.

20 The text from the headings to chapters 2 to 32 is quite closely copied by Syrianus in his commentary on Aristotle's *Metaphysics*, 101,26–102,35, translated by John Dillon and Dominic O'Meara in the series Ancient Commentators on Aristotle.

Against Aristotle's downplaying of mathematical objects as abstracted from matter by our own minds, Syrianus takes Plato to have been following the account of mathematical objects by the ancient Pythagorean Archytas of Tarentum, who is quoted in *DCMS* chs 8 and 11. But much was attributed to Archytas by the post-Platonic Pseudo-Archytas. RRKS

21 *theôria*. RRKS

22 *horos*: I believe this can mean definition, standard, or division, according to context. RRKS

23 *horos*: as often, Iamblichus' opening signposting of the chapter's contents (sometimes of its initial contents followed by signposting of its later phases) matches the wording of the chapter headings. Often the main contents are unreferenced quotations, but in this chapter Iamblichus appears to me to be chiefly stating the questions to be addressed, with an explicit reference to Archytas, rather than an unreferenced quotation. RRKS

24 Being delimited, as opposed to being unlimited, is a merit, since the unlimited lacks form.

25 *logoi*, *eidê*. RRKS

26 Cf. Plato's simile in *Republic* 509D–511D of the divided line, representing four different states of cognition, the subject of chs. 7 and 8 below. The Pythagorean mathematician and general, Archytas of Tarentum in southern Italy, *c*. 470–*c*. 385, was an older contemporary of Plato and rescued Plato from the Sicilian tyrant Dion (Diogenes Laertius, *Lives of Eminent Philosophers* 8.79). Iamblichus takes him to have preceded Plato in discussing the division of a cognitive line. See further ch. 8. RRKS

27 This much at least is Iamblichus' editorial comment. RRKS

28 See the notes on ch. 8 below for discursive reasoning. RRKS. And see ch. 8 itself for a long quotation attributed to Archytas' treatise. SG

29 Reference to Aristotle's thinking that the objects of mathematics are the product of our minds abstracting from the inexactness of material things. This is alleged to make the objects of mathematics changeable like the material things.

30 This view that the soul is a further kind of substance, over and above Plato's ideal Forms, mathematical objects, and sensibles is ascribed to Plato's nephew and successor, Speusippus, by Aristotle, *Metaphysics* 1028b21–24 – so P. Merlan, *From Platonism to Neoplatonism*, 1960, pp. 96–7. The next chapter, 4, is largely concerned with Speusippus' views. RRKS

31 13,17–18. These principles of the unlimited and limit were used by Plato in his *Philebus* in describing a number of things from acoustics to the cosmos. In ch. 7, 29,20 their origin is ascribed to the Pythagorean, Philolaus. They were later used after Plato by the Neopythagoreans, whose application of them to the

mathematical sciences is discussed below in ch. 7, 28,24–32,7. They are cited at ch. 1, 11,14; ch. 3, 12,24 and 13,17–18; ch. 9, 41,5; ch. 11, 44,1; ch. 12, 46,2; ch. 13, 49,8 and 50,13; ch. 18, 62,23; ch. 20, 66,2. RRKS

In ch. 7, 28,24–32,7, the ideas of discrete and continuous are described in terms of limit. Hutchinson compares the discussion, well known to Iamblichus, in Nicomachus of Gerasa's *Introduction to Arithmetic*, esp. 1.2–3, of which Iamblichus' fourth title in his Pythagorean sequence is an edited version. From DSH

32 Add. Steel, to secure an acceptable sense.

33 This at least is Iamblichus' editorial comment. RRKS

34 An application of the well-known Aristotelian formulation for logical priority and inferiority, e.g. Aristotle, *Categories* 7b19 ff; *Metaphysics* 1059b31ff.

35 Iamblichus' introduction appears to end here. The core of the chapter presents quotations from Plato's nephew Speusippus about the status of mathematical objects deriving from a 'One' and from a material principle of plurality – so Merlan, *From Platonism to Neoplatonism*, 1960, p. 98. Indeed, Merlan thought that some, perhaps most of ch. 4 might be a literal quotation from Speusippus, p. 120, although see the note below on 18,1 ff., where he saw the last part of the chapter as summary rather than quotation, and he added in this second edition that the chapter was not everywhere a photographic copy of the corresponding text in Speusippus, p. 131. RRKS

36 It has been conjectured – credibly, in my view – by Philip Merlan that what follows, up to the end of the chapter, is derived, perhaps through some intermediary (Neopythagorean?) source, from a work of Speusippus on the first principles, since the metaphysical scheme outlined here seems to correspond only to that elsewhere attributed to Speusippus. Cf. also Dillon, 'Speusippus in Iamblichus', *Phronesis* 29 (1984), 325–32, and 'Reconstructing the Philosophy of Speusippus: A Hermeneutical Challenge', in Ulrike Bruchmüller, ed., *Platons Hermeneutik und Prinzipiendenken in Licht der Dialoge und der antiken Tradition: Festschrift für Thomas Alexander Szlezák zum 70. Geburtstag* (Hildesheim: Olms Verlag, 2012), pp. 185–202.

37 Merlan, *From Platonism to Neoplatonism*, 1960, p. 104, interprets Speusippus as meaning that it is above being, as later in Plotinus. RRKS

38 This seems to be a devious allusion to the 'persuasion' of Necessity by Reason in *Timaeus* 48A; and cf. also *Sophist* 267d7: *meta peithous anangkaias*.

39 Merlan, *From Platonism to Neoplatonism*, 1960, pp. 110–15 takes the principle called the 'One' not to be inferior to the good, even though the good (and the beautiful) and their opposites are not found in that sphere of being, which is above (16,11) all that, nor are they in the sphere of being of Speusippus' plurality principle. Beauty first appears in the sphere of numbers, as at 16, 4–5 and 18, 5 below. RRKS

40 This is an argument against assigning goodness or 'fineness' (*to kalon*) to the One, such as is attested for Speusippus, here going against his uncle Plato, cf. Speusippus, frr. 42–3 Tarán. This is developed further below.

41 That is to say, (probably) when the elements are at a further remove *from one another*, by reason of extension.

42 A marginal note on this passage suggests that Iamblichus is here introducing the notion of 'intelligible matter' (*noêtê hulê*), presumably because such a concept would explain the differentiation of immaterial entities after the One. SG

43 Perhaps 'solid figures', as at 17,27.

44 None of which will represent the figure in exactly the same way.

45 In other words, Speusippus thinks that the unity of the One is still represented in the monads (or units) of which each number is composed, and in geometrical points, and (perhaps) in the lines, areas, and solids from which extension and position are derived. RRKS

46 A scholium here states: 'the baseness and filth of numbers arose from the weak matter of magnitudes,' showing here at least a dim understanding that there are different levels, or modes, of matter involved. JMD. But Merlan, *From Platonism to Neoplatonism*, 1960, pp. 120–1 takes *summemolusmenon* in line 17,20 in a different sense as meaning not tainted, but under-done, i.e. not sufficiently mastered by the formal principle of the One. RRKS

47 John Dillon, 'Speusippus in Iamblichus' *Phronesis* 29 (1984), 329, agreed with Merlan that this last part of ch. 4 appears to be summary rather than quotation. RRKS

48 The elements in question, if this is drawn from Plato's successor Speusippus, will be his One and principle of plurality. The latter may itself be a successor of sorts to the indefinite dyad ascribed by Aristotle to Plato. RRKS

49 The reference here would seem to be to the realms of animate and inanimate physical objects, or possibly rational and irrational living beings. The level above them, and below that of the geometricals, namely Soul, has been omitted, as not germane to Iamblichus' present purposes. We have here, then, a five-level universe, from which the first principles, One and Plurality, are excluded, qua supra-essential first principles.

50 Here begins Iamblichus' summary of the chapter, so Merlan, *From Platonism to Neoplatonism*, 1960, p. 98.

51 This should properly refer to the level of Soul, but Iamblichus seems to be referring rather to the level of embodied living beings.

52 Here Iamblichus concludes his report of Speusippus' account with an endorsement. RRKS

53 Syrianus, in his paraphrase of this particular heading, replaces 'students of mathematics' with 'Pythagoreans', which may well be what Iamblichus intends here. See *in Metaph*. 101,36. SG

54 Or lovers of mathematics: the Pythagoreans; cf. Syrianus, *in Metaph*. 101,36. RRKS

55 At 19,4 the text has *theôreitai*. I accept the editor's *theôrei kai to*.

56 19,7: *epibolê*. RRKS

57 The text is untranslatable at this point, 19,9. The editor suggests the addition of several words to complete the sense, and we have translated those.

58 This appears to me to be the end of Iamblichus' opening orientation. RRKS

59 Iamblichus' concluding editorial comment. RRKS

60 The present chapter is composed of a cento of extracts, first from Pseudo-Plato, *Epinomis*, then from Plato, *Republic*, Book VII. In more detail: 20,22–21,15 = *Epinomis* 991D-992B; 21,15–20 = 991BC; 21,21–22,5 = 986CD; 22,5–11 = *Rep.* 537CD; 22,13–19 = 536B; 22,19–23,4 = 527DE; 23,4–10 = 521CD; 23,10–28,14 = 523A–532D *passim*. JMD

Here the references to Plato and the Pseudo-Platonic *Epinomis* start right away, replacing any editorial comment by Iamblichus, and Iamblichus' concluding editorial comment is a very brief sentence. His treatment of the passages is parallel to the earlier use made by Nicomachus of Gerasa in *Introduction to Arithmetic* 7,9 ff., a text of which Iamblichus' fourth Pythagorean title was to be an edited version. From DSH

61 *to kath' hen*: the unitary, hints at the influence of the One. RRKS

62 The Pythagorean approach takes mathematics to explain the structure of the cosmos. P. Merlan (1960), p. 148, points out that this sentence inserts the Pythagorean idea between two curtailed sentences of the Pseudo-Platonic *Epinomis*, which do not say this at all. RRKS

63 This is a reference to Pythagoras.

64 See Plato, *Republic* 521C5 for this image. SG

65 This down to 25,13 is a version of Plato, *Republic* 523A10–524 D5. RRKS

66 *logistikê kai arithmêtikê*: we have so translated these terms. But the former covers most of what we now count as arithmetic, while the latter term denotes number theory rather than arithmetic.

67 This whole parable about release from the cave up to the dazzling Forms, headed by the Good, which at first can be seen only in shadows and reflections, comes from Plato's *Republic*, as recapitulated in *Republic* VII 532B-C, together with Plato's simile of the four-part line, representing different levels of reality and our experience of them, is the background to most of this work.

68 28,14: 'visible', reading with ms P, *horatōi*. Festa's text has *ahoratōi*.

69 Here ends Iamblichus' introductory comment, followed by seventy-five lines not by Iamblichus, but quoted by him from a source or sources. RRKS

The seventy-five lines from 28,24 to 32,7 (Festa) are exactly reproduced again by Iamblichus as they occur in this, the third title of Iamblichus' Pythagorean series, in his fourth title, *Commentary on the Introduction to Arithmetic* of Nicomachus of Gerasa 7,3–9,23, Pistelli. In that fourth title, Iamblichus no longer separates his quotations off into annotated chapters, but melds them with the *Introduction* of Nicomachus (for which see the English translation and notes by D'Ooge, 1926). Thus Iamblichus has used the quotation in two works. From DSH

The first nineteen lines of this overlapping passage, (28,24–29,18 Festa) were roughly reproduced also in the commentary on Aristotle's *Categories*, 66,23–67,1 by Iamblichus' younger associate, Dexippus, translated in this series by John Dillon, and would much later be again, though more loosely, in Simplicius' commentary on Aristotle *Categories* 126,6–10 and 126,23–127,11. Dexippus has been thought to draw on the lost commentary of Porphyry (Pierre Hadot) or primarily on Iamblichus' commentary (Concetta Luna), as recorded in the second edition of Richard Sorabji, *Aristotle Transformed* (London: Bloomsbury, 2016), pp. xx–xxiii.

70 Reading at 29,1 *pasi*, *pasa* in the text, corrected in the apparatus.

71 Plato, *Philebus* 16C speaks of a gift from some Prometheus according to which whatever is said to be consists of one and many, having in its nature limit and unlimitedness. Carl Huffman has taken this to be an acknowledgement of the Pythagorean Philolaus, 'The Philolaic Method: The Pythagoreanism behind the *Philebus*', in A. Preus, ed., *Essays in Ancient Greek Philosophy VI: Before Plato* (New York: SUNY Press, 2001), pp. 67–85. RRKS

72 The superparticular number (*epimorios*) has the value $1 + 1/n$, $n > 1$; e.g. $1 + 1/2$. The superpartient (*epimerês*) has the value $1 + m/n$, $m > 1$, $m < n$; e.g. $1 + 2/3$.

73 'the two above-mentioned sciences', i.e. arithmetic and geometry. SG

74 Archytas, *On Mathematical Science*, fr. B1 in Diels-Kranz, *Fragmente der Vorsokratiker*, preserved in Porphyry's commentary on Ptolemy's *Harmonics*. RRKS

75 This and the next seven lines are a recognizable paraphrase embedded within Iamblichus' citation of the unknown seventy-five line quotation. The paraphrase is of *Epinomis* 992A–B, which the larger source ascribes to Plato, whom it considers 'divine', as did Nicomachus in his *Introduction to Arithmetic*. Chapter 3 also cites the embedded passage from *Epinomis*. But while neither Iamblichus nor Nicomachus cite the next words of *Epinomis* which express a concern for happiness or flourishing, Iamblichus here does go on immediately to expect that philosophy will lead to flourishing. From DSH

76 4,12: *kritêrion*. This word occurs in the chapter heading and in ch. 8, at 32,8; 35,24; 36,1 and 2; 39,19 and 23; 40, 6; and ch. 29 at 90,8; 98,2. The term is derived from

krinein, to make judgements, and is connected in ch. 8 below, with Plato's division in his *Republic* 509D–511E of a line into four segments corresponding to four experiences (*pathêmata*) of the soul: the intellection (*noêsis*) of the philosophical dialectician, the step-by-step reasoning (*dianoia*) of the mathematician, based on hypotheses and diagrams, unreliable trust (*pistis*) to do with perceptible things and hearsay, and the use of images (*eikasia*) directed at mere reflections of the visible. Iamblichus refers to these four as capacities which make judgements (*krinousai dunameis*) at 39, 27. But at 39, 19–24 he picks up the word *kritêrion* from earlier in the chapter, and ascribes to the philosophical dialectician the *kritêrion* of intellection, *noêsis*, and to the mathematician the criterion of step-by-step reasoning, *dianoia*, using hypotheses and images. I am not clear that the *kritêrion* here is simply identical with each of the four most general judging capacities in the divided line. In the case of step-by-step reasoning (*dianoia*) used by mathematics, there are a lot of sub-capacities included within that general capacity, for example the capacity for the use of hypotheses and images just mentioned as used by the mathematician, and the analysis and synthesis ascribed to the mathematician in ch. 17, at 59,7 and ch. 20 at 65,15, on which see the notes. If the criterion in this case includes capacities for certain particular methods of reasoning used for arriving at a judgement, this will fit well with the idea in ch. 29 at 90,8–9 that a criterion can be used for 'approving for oneself' (*dokimazesthai*) a definition.

For a succinct history of the philosophical use of the term *kritêrion* up to the great astronomer Ptolemy's *On the Criterion* and Sextus Empiricus' sceptical attack on dogmatists in Book 7 of his *Against the Mathematical Scientists*, both of the century before Iamblichus' reported date of flourishing, *c.* 290–320 CE, see A. A. Long, in 'Ptolemy on the Criterion: An Epistemology for the Practising Scientist', in P. Huby and G. Neal, eds, *The Criterion of Truth* (Liverpool: Liverpool University Press, 1989), pp. 151–78. The idea of a criterion became important in the Hellenistic period, he argues, dated from a year after Aristotle's death, 323 to 31 BCE, in the context of a sceptical debate as to whether there was any infallible criterion for reaching the truth. But Iamblichus does not have to address these later worries about infallibility very closely, because he is quoting the views from the earlier period of Plato and of Archytas the latter of which he takes to be embedded in the account in Plato's *Republic* of the divided line with its four faculties, and of the cave from which the Guardians of Plato's ideal republic can escape, thanks to an education starting with the senses, continuing into the mathematical sciences, followed by practise in the dialectic of philosophical debate, and then ordinary political experience, after which the Guardians, escaping the cave, can suddenly see, through the highest faculty, intellection, what true

goodness is, and so make secure political judgements back down in the cave of ordinary political life. RRKS

77 The cognitive line is a central subject of the chapter. See the note above on 12,12–13, where Plato's older benefactor, the Pythagorean Archytas of Tarentum, is said to have discussed division of the cognitive line, which might suggest he influenced the use of a divided line in Plato's *Republic* 509D–511D, to illustrate the relations between four cognitive states. But below at 36,1 ff., where Iamblichus quotes Archytas on the divided cognitive line from his treatise *On Intellect and Sensation*, this is not included by Diels-Kranz, *Die Fragmente der Vorsokratiker*, among the genuine fragments of Archytas. And 36,3 ff. looks suspiciously like a paraphrase in Doric dialect of Plato, *Republic* 509D ff. Many Neopythagorean fragments and testimonies under the name 'Archytas' have been considered inauthentic. Plato uses his own discussion of the line to support his proposals on education to enable future rulers to achieve the highest cognitive states first through the mathematical sciences, and then by philosophical argumentation, 521C–541B. RRKS

78 Iamblichus starts many of his chapters, as here, with a restatement of what the chapter is about, using words partly different from those in the italicized chapter heading, but heading and restatement use the same words where they are considered, like *kritêrion*, as keywords for describing the chapter. RRKS

79 32,13–16: discursive thought (*dianoia*) is step-by-step reasoning. Intellect (*nous*) is superior and associated rather with steady insight, that may, however, start with a sudden realization. From 32,16–33,19, the text distinguishes two lower cognitive states, trust in the senses or hearsay called trust (*pistis*) or belief (*doxa*), connected with *dokein*, 'to seem', and, lowest of all, reliance on mere images (*eikasia*). RRKS

80 Earth, air, fire, and water. RRKS

81 33,13–14: *doxasta*. Here Iamblichus reverts to the other name, objects of *doxa*, 'belief', instead of 'objects of trust', *pistis.* But he compensates when he reminds us of the insecurity of *doxa* by connecting it with *dokein*, mere 'seeming'. RRKS

82 33,22: *epibolê*. RRKS

83 Abstraction: *aphairesis*. This dismisses Aristotle's mature view, not yet expressed in his *Protrepticus*, that the objects of mathematics had a lower status than that accorded by Plato. A geometrical circle is abstracted by our minds from the imperfections of roughly round material objects like wheels. RRKS

84 Accepting a conjecture of Festa.

85 This and another reference by Iamblichus to Bro(n)tinus are included by Diels-Kranz, *Die Fragmente der Vorsokratiker*, as reports of a pre-Platonic Pythagorean. RRKS

86 Adopting the proposal of Festa to insert *apodeiktikon* after the otherwise meaningless *ton*.

87 Clearly not logically contraries. A scholiast says: 'For in things power is inversely related to plurality.'

88 This last clause is a guess at the approximate sense; the text, which is untranslatable, reads *tauta sumphona poiêta theôroumena di'autôn alathea*. In effect we read *poiei ta theôroumena*, with Vitelli.

89 Reading *kathaper gar gramman dikha tetmamenên anisa tmêmata kai palin hekaterôn tmêmata tetmêmena ana ton auton logon.* The text reads *kathaper gar gramman dikha tetmamenên kai isa palin hekaterôn tmêmata tetmamena ana ton auton logon*, which translates as: 'for like a line cut in two and again equal segments cut from each in the same ratio'. But no ratio is specified. We need segments varying in length to represent intellect, discursive reasoning, trust and imagination by decreasing lengths. My text is derived from that of Plato, *Republic* 509D.

90 Reading, with Festa, [*ouk arkhas*] *all' hormas te kai epibasias*. The text reads untranslatably: *hypothesias all' hypothesias all' arkhas te kai epibasias*. Once again see *Republic* 511B.

91 Lines 38,1–6 are reproduced without significant changes by the thirteenth- to fourteenth-century commentator on Aristotle, Sophonias, *in DA* 130,29–32. SG

92 Cf. reference back at 98,2–3.

93 Mathematicians, rather than mathematical objects, thus providing the contrast with the dialectician below. RRKS

94 I can attach no meaning in the context to *prognostikon*, 'prescient', and so translate simply *gnóstikon.* In the manuscripts, the *pro* seems to be written above the line.

95 That is, this (intellection) is the criterion, or *capacity for studying* used by the philosophical dialectician, and through this (intellection) he (the dialectician) both views . . . RRKS

96 A scholium reads: 'dialectic' is to be understood not as the Peripatetic [Aristotelian] kind which aims to lead the respondent into contradiction but the Platonic kind, or rather the Pythagorean. For Plato learnt it from the Pythagoreans. This dialectic is the cognition of things that have being as they are, and it uses no hypotheses.

97 Lines 39,26–40,4 are reproduced without significant changes by the thirteenth- to fourteenth-century commentator and paraphrast, Sophonias, *in DA* 130,33–5. SG

98 The text reads *idômen prôtên doxan*. I translate *idômen prôton tên doxan.*

99 Iamblichus' editorial introduction asks how close is the relation of mathematics to the soul, a subject continued in ch. 10. RRKS

100 40,14: *epibolê*. RRKS

101 Merlan, *From Platonism to Neoplatonism*, 1960, p. 9, says that this amounts to a quotation from Speusippus. RRKS

102 This view is attacked by Aristotle in his *De Anima* (*On the Soul*), 1.4, 408b30–1.5, 409b18. RRKS

103 The first two definitions are those of Plato's immediate successors in the Academy, Speusippus and Xenocrates respectively; the third may be that of the Neopythagorean Moderatus of Gades, or possibly even that of Plotinus, to judge by Iamblichus' listing of definitions of the soul in *De Anima* 4–5, 364–5. Cf. J. F. Finamore and J. M. Dillon, *Iamblichus' De Anima*, *Text*, *Translation*, *and Commentary*, (Leiden: Brill, 2002), p. 28.

104 For the Pythagorean use of the idea of the limited, see ch. 3 above. RRKS

105 The Pythagoreans were disconcerted to find that some numbers were *irrational*, that is, not expressible as the ratio of two whole numbers, for example the ratio borne by the length of the diagonal to the side of a right-angled triangle. Two numbers are *commensurate* if their ratio is so expressible as the ratio of two whole numbers. RRKS

106 This chapter, like its predecessor, to judge from the prose style as well as the content, seems solidly Iamblichean in provenance. It rounds off his discussion of the essence of mathematics.

107 Reading *apogennômenê* instead of *apogennômenês*.

108 This and the preceding chapter (40,12–43,11) are reproduced with only minor changes in Sophonias, *in DA* 131,11–132,24. SG

109 The main source for this chapter is Plato. The one passage, attributed to the Pythagorean Archytas, is probably from the Neopythagorean Pseudo-Archytas. RRKS

110 Scholium: 'from sensible objects'.

111 Throughout this chapter the Platonic theory that mathematical proof is the recollection of what one formerly knew before birth is presupposed, for which Plato, *Phaedo* 72E–78B supplies a major source.

112 An application here of 'Meno's Paradox' from Plato, *Meno* 80D: how can the unknown be recognized if you don't know it already?

113 'the number of change' is an Aristotelian definition of time, *Physics* 4.11.

114 50,3: *kath'hauto ousia*. RRKS

115 50,10–11: *hupostasis*, a Platonist term for levels in a hierarchy of beings, also at the heading of ch.10 and 39,2; 39,11; 42,17. RRKS

116 An example of this is given in the note on analysis and synthesis in ch. 17, at 59,8, which envisages proving the Pythagorean theorem that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the

other two sides, by first constructing a diagram showing the equality of the one square to the two. RRKS

117 Scholium: 'Meaning that similarity in mathematics is so called homonymously with (*sc*. in a different sense from) the similarity found in the cases of things intelligible and things sensible.'

118 Scholium: 'Because similarity between complexes is viewed qualitatively, and as incidental, but in the case of mathematical objects similarity and dissimilarity are essential.'

119 Translation of 'substance' (*ousia*) is difficult in ch. 14. For Iamblichus, mathematics and its objects (contrary to Aristotle's rejected theory that its objects are mere *abstractions* made by our minds from inexact material bodies), belong to Aristotle's category of substance (*ousia*), as opposed to such other categories as quality, quantity, etc. Since it is Aristotle's categories that Iamblichus is here discussing, we probably have to translate that use of *ousia* as 'substance'. But mathematics and its objects are also said to *have* an *ousia of* themselves. For this usage it would be too confusing to abandon the translation elsewhere, and it is best to keep for it the translation 'essence' or 'being'.

In certain other contexts *ousia* can be translated differently as 'existence', but with that too there are obstacles, because at 33,19, for example, it is reality, not existence that is in question. RRKS

120 Scholium: 'Similarity and dissimilarity are meant as in a category; similarity in quality as white is similar or dissimilar to white and hot to hot; that arising from figure as a material triangle is similar or dissimilar to a material one; in relation of position, as Byzantium is similarly situated to Sinope [. . .]; in mathematical substance, as to describe a solid parallelogram as similar to, and similarly situated as, a given solid plane parallelogram. Because similarity and dissimilarity predicated of mathematical objects is substantial and a form of mathematical substance; for their likeness is not incidental, since, being immaterial, it could not accept the discord and sordidness of incidental features.'

121 Scholium: 'That it is necessary in the case of similarity and dissimilarity of mathematical objects to regard the unit generically, plurality as of species and the intermediate in due order, and thus to fit to each mathematical substance its appropriate similarity and difference.'

122 This looks like Iamblichus' summing up after drawing on sources which need to be identified. But relevance to the affairs of life and divine and human goods does not seem to be explained in this chapter as it is in the next two, chs 15 and 16. (For life, cf. also chs 22, 23, 26). Is the comment uncharacteristically announcing the subject of the next two chapters, or even chs 15–19? RRKS

123 Proclus provides a fairly free adaptation of the whole of ch. 15 in his *Euclid Commentary*, 20,8–24,20.

124 In this chapter, Iamblichus argues for the wide utility of mathematics to philosophy, in terms of its psychological effect. It draws the mathematician from physical to intelligible beings and to theology. But it also indicates (*paradeiknunai*, 55,24) to the natural scientist the analogy (*analogia*, 55,25) between the natural world and the mathematical. It provides mathematical models or paradigms (*paradeigmata*, 56,12.17) for the statesman (56,4) and teacher of ethics and morality (56,8). By its similarity (56,25) to the parts of philosophy it can lead us from the forms in matter to the unchanging Forms that are superior to it. RRKS

125 That is to say, using the orderliness of static theorems as a model for orderly laws of practical interaction. RRKS

126 Thus we are told that the Pythagoreans identified justice with the square, or else the pentagon. JMD. But the present account suggests that the identity they postulate is a matter of an inspiring analogy. Cf. also ch. 18, 61,1 ff.; ch. 19, 63,27. RRKS

127 Chapters 16 and 17 are argued by T. Orlandi, 'Il primo Aristotele nel De communi mathematica scientia di Jamblico', *Rivista critica di storia della filosofia* 18 (1963), 592–8, to have been taken from an early work of Aristotle, either directly or via some epitome. D. S. Hutchinson has reviewed this area of the *DCMS*, and finds this thesis of Orlandi very viable, and highly compatible with the Hutchinson-Johnson understanding of both Iamblichus' *DCMS* and Aristotle's *Protrepticus*. From DSH/MRJ

128 There is always a difficulty about the accurate rendering of *tekhné*, but this seems to be the best rendering in the present context.

129 See the first note to ch. 16, on T. Orlandi's arguing that chs 16 and 17 have been taken from an early work of Aristotle, either directly or via some epitome, and D.S. Hutchinson's approval of this thesis as very viable and highly compatible with the Hutchinson-Johnson understanding of both Iamblichus' *DCMS* and Aristotle's *Protrepticus*: T. Orlandi 'Il primo Aristotele nel De communi mathematica scientia di Jamblico', *Rivista critica di storia della filosofia* 18 (1963), 592–8. From DSH/MRJ

130 The Greek mathematician Pappus of Alexandria (early-fourth century CE) gave the standard, but oversimplified, account of analysis and synthesis in Greek geometry in his *Treasury of Analysis*, Book 7. He spoke of analysis as a *reverse* solution (*anapalin lusis*) and as if synthesis were its *reversal,* to be used as a counterpart of analysis. But J. Hintikka and U. Remes, *The Method of Analysis: Its Geometrical Origin and its General Significance* (Dordrecht: Reidel, 1974) offered

a fuller reconstruction, to get round such difficulties as the idea of simple reversibility. To prove by analysis Pythagoras' theorem cited here at 59,22–4, that the square on the hypotenuse of a right-angled triangle is the sum of the squares on the other two sides, one would need to draw the triangle and its three contiguous squares and see by analysis where to place auxiliary lines to form congruent triangles within the figure, and where to find existing theorems about congruent triangles. Then from those triangles and theorems one could *reverse* and prove, by a synthesis assembling the diverse data, the equality of the two squares to the other one. However, at 65,15–20 below, a much simpler account of analysis and synthesis is ascribed to the Pythagoreans in the different context of making definitions. Analysis involves dividing, as also at 10,13 and 20,11 and, in the case of definitions, dividing into genus, differentia, and species. Synthesis here involves an 'assembling (*sunagôgos*) composition (*sunthesis*)' of a definition out of the previously divided components. There is an analogy between the complex geometrical example and the simpler definitional one: in both cases there occurs a reversal such as that of which Pappus spoke and Hintikka and Remes allow for. It seems safer to use the translations 'composition', 'compounding', for *sunthesis* in this context, rather than 'synthesis', because the word is repeatedly used in this everyday sense in other parts of the text. But we cannot overlook that at 59,8 the Greek terms *sunthesis* and *analusis* are applied to mathematical procedures much more complicated than the definitional procedure described at 65,15–20.

However, there is an analogy (see the note on 65,15–20) in that a *reversal* occurs in both the complex and the simple procedures. In the simple one, reversal is indicated in the words, 'in both directions mathematics [25] must study the unitary one from which it begins *and to which it is led back*' (65,25–6). And it is carefully pointed out that mathematicians do use the simpler definitional procedure as well (65,7–11). Moreover, in both examples there is a process of dividing things up and then assembling them. RRKS

131 Cf. 62,11: *epibolê*. RRKS

132 But Aristotle(?), *Magna Moralia* 1182a11–14 says that justice was, according to Pythagoras, a number that was *isakis isos*, i.e. equal multiplied by equal or four-square.

133 This is not made so clear for the case of the original Pythagoreans as for the Neopythagoreans in the wake of Plato's Academy. RRKS

134 62,11: *epibolê*. RRKS

135 A glancing reference to Xenocrates' definition of the soul as 'number moving itself'. JMD. For Aristotle's attack on it, see his *On the Soul* 1.4, 408b30–409a30. RRKS

136 *Sc*. of the mind from material concerns and confusions.

137 Has Iamblichus' comment got misplaced? This looks more like a reference forward to the present ch. 19. RRKS. An alternative would be that this is a comprehensive reference to the various treatises later in the series devoted to mathematics in relation to various other disciplines; cf. 76,14 below.

138 See the note on the reference to mathematical analysis and synthesis, with its geometrical example at 59,8. This new passage at 65,15–20 is very interesting. It starts by using the Greek word *sustasis*, but apparently in the sense which it has often had in the text, which is now called an 'assembling (*sunagôgos*) composition (*sustasis*)' in contrast with a division. The point being made at 65,7–11 is that one of the tasks of mathematics is to provide *definitions*, and definition involves a prior process of first dividing some unitary subject into genus, differentia, and species, and then a reverse process of assembling a composition of genus, differentia, and species to provide a definition of the species. This *reversal* is part of what makes it possible at 65,15–20 to move from contrasting the *sustasis* with division, to contrasting it as a *synthesis* with an *analysis*. Analysis itself involves dividing, as remarked also at 10,13 and 20,11. But this was harder to see in the much more complex example of geometrical analysis at 59,8. That example did involve the division of a figure by drawing auxiliary lines within it. But the more obvious analogy with that complex example is that in both cases there was a *reversal* such as Pappus required in analysis and synthesis. One procedure divided the figure, and then, with the aid of extra theorems, one was able to reverse and prove what started as a mere assumption about the figure. It seems safer to use the frequent translations 'composition', 'compounding', for *sustasis* at 65,10, in the opening lines, from 65,7–15, rather than 'synthesis'. But from 65,15–20, where analysis is mentioned twice, we realize that this composition, so far contrasted with division, is also a case of synthesis as opposed to analysis. One ground for this is the *division* involved. But another ground is the presence of *reversal*, emphasized at 65,25–6 when it says, 'in both directions mathematics [25] must study the unitary one from which it begins and to which it is led back'. It looks as if there is only an analogy between different cases of analysis and synthesis. Analysis involves various forms of dividing, which are followed by various forms of compounding or assembling, after a *reversal*. RRKS

139 This heading covers ch. 22 as well. From DSH/MRJ

140 Thales of Miletus, often viewed as the earliest Greek philosopher/scientist, caricatured by Iamblichus in his *Exhortation* ch.14 as the absent-minded philosopher, on the basis of the story in Plato *Theaetetus* 174A that he was laughed at by a witty slave girl for falling down a well while contemplating the stars. The firmest date is 585 BCE for an eclipse which he predicted. RRKS

141 This may be the wording of Iamblichus, keen to associate Pythagoras with the famous philosopher/scientist, rather than quotation from a source. From DSH/MRJ

142 Aristotle could be the source of this information, since in work later than his *Exhortation* he says that Egyptian priests invented geometry (*Metaphysics* 1.1, 981b23) and that Egyptian and Babylonian records were the source for much of the evidence about particular stars, *On the Heavens* 292a8, cf. 270b1–25. From DSH/MRJ

143 Diogenes Laertius, *Lives of Eminent Philosophers* 1.1 says that Aristotle, in a (lost) work called *Magicus*, said that the Persians have their Magi, the Babylonians or Assyrians their Chaldaeans. From DSH/MRJ

144 Hutchinson and Johnson in their 2017 online reconstruction of Aristotle's *Exhortation*, at http://www.protrepticus.info/protr2017x20.pdf take 67,7–12, 67,16–17, and 67,19–68,4 to be quotations, interrupted by Iamblichus' comments, from the speech put into the mouth of Heraclides in Aristotle's *Exhortation to Philosophy*. From DSH/MRJ

145 A reference to the use of so-called 'symbols' (*sumbola*) in Pythagorean circles; cf. Iamblichus, *On the Pythagorean Life* 18 and *Protrepticus* 18. SG

146 On Iamblichus' parallel belief that the priestly power of the theurgic priest depends on knowing the natural names of the gods, see Sorabji's introductory essay in this volume. RRKS

147 Omitting the *to* before *on,* suggested by Festa.

148 *to hupsilonoun*, literally high-mindedness. RRKS

149 The Greek words corresponding to the English 'for the present' (or 'for the time being') could also be rendered as 'in the present day', thereby suggesting that Iamblichus recognized the elements of his idealized conception of mathematics in the contemporary practice of that science. Such a reading, however, would be incompatible with Iamblichus' explicit criticism of contemporary mathematics at 69,22–4. SG

150 68,28: 'mathematical being' (*ousia*) or 'essence'. See the note on ch. 14, at 51,27, which has the same expression. RRKS

151 The heading here, unlike Iamblichus' opening of the chapter a few lines below, does not do full justice to the diversity of opponents whom the chapter will discuss. His point is that the value of mathematical philosophy is not *solely* for the benefits it brings to daily life, as some of its protagonists had maintained. But neither, as some antagonists had urged, is it useless for daily life. Rather (71,16–17), it has been a help to us already for many things necessary for living and for optional extras and also for things that are valuable for their own sake. Again (71,26–72,4), one would not cherish its power *only* on account of the help it gives,

but, still more, for itself and its own nature. For it is agreed that some sciences are choiceworthy for their own sake and not *only* for the results they bring about. This is further confirmed in ch. 26, 82,11–13 and 84, 7–11. RRKS

152 In 'A new fragment of Aristotle', ch. 6 in his *Platonism and Neoplatonism*, 1960, Merlan says (p. 144) that *DCMS* ch. 23 is 'either a series of quotations or one quotation from Aristotle'. He is referring to Aristotle's early work, his *Exhortation to Philosophy* (*Protrepticus*), in which Aristotle is still under the spell of his teacher Plato and Plato's Academy in their estimation of the status of mathematical objects, before he had reached his mature view that mathematical objects are abstractions drawn by our minds from sets of material objects and their imperfect edges, surfaces and shapes. Abstraction is mentioned in chs 8, 22, 28, and 32. RRKS

153 Iamblichus takes Pythagorean philosophy not to be separate from mathematical matters (*mathêmata*). RRKS

154 'These' are probably the things revealed and demonstrations of Pythagoras in lines 3–4. RRKS and SG

155 'This field', literally 'these things', probably refers to the *mathêmata* or mathematical sciences used by philosophy in line 2, but these will include study of the cosmos. RRKS

156 The methods are not those of empirical investigation (70,14), or of familiarity with particular instances, and can therefore be undertaken in youth (70,24–5). RRKS

157 There is a lacuna in the text. Presumably some such phrase as *touto de tais* is missing.

158 'Philosopher' is a name composed of the Greek words meaning lover of wisdom. RRKS

159 As Merlan points out (p. 142), *DCMS* ch. 23, including the passage at 71,16 ff. is a response to the opponents of philosophy as a theoretical science who also oppose the mathematical sciences on the ground of their alleged uselessness to the activities and function of life as producing good and useful things. These opponents are described by Iamblichus below in ch. 26, 79,1–84,20. Merlan knew that two passages which Iamblichus quotes in ch. 26 had already been identified as fragments drawn from Aristotle's *Protrepticus*. Merlan argues (pp. 142–3) that the replies in ch. 23 can therefore reasonably be taken to be drawn from Aristotle's *Protrepticus* as well. He adds that *DCMS* ch. 23 is similar to chs 6 and 9 of another work of Iamblichus, his *Protrepticus* or *Exhortation to Philosophy*. And these Iamblichan chapters in turn contain two excerpts from Aristotle's own *Protrepticus*, frr. 5 and 13 in Walzer's reconstruction. RRKS

160 Merlan (pp. 151–2) draws attention to Isocrates' role as opponent of mathematical studies. An inexact knowledge of useful things is more important

than an exact knowledge of the useless (*Helen* 5). Mathematics is not a philosophical study, but an elementary propaedeutic (*Antidosis* 266–9). It merely keeps young boys out of mischief (*Panathenaicus* 27). Hutchinson and Johnson have reconstructed Aristotle's *Exhortation to Philosophy* provisionally in 2017, www.protrepticus.info/protr2017x20.pdf, as including an imagined dialogue between Aristotle himself, Isocrates of Athens, as a teacher of philosophy and an opponent of mathematical philosophy, and Heraclides of Pontus, Aristotle's fellow student under Plato, as an unqualified supporter of mathematical philosophy and the Pythagoreans. Iamblichus' quotations from here may represent the views of any of the three imagined interlocutors. RRKS

161 Merlan (pp. 143–4) gives another reason for taking Aristotle to be Iamblichus' source for the argument that disciplines gain their value from their exactitude and the value of their objects. It is that this argument survives in the opening sentence of Aristotle's much later work *On the Soul*, at 1.1, 402a1–4, as an unexpected justification for the study also of *psychology*. RRKS

162 *astrologikê* is the term used by Aristotle for 'astronomical science', one piece of evidence that this passage is being quoted from Aristotle. The term *astronomia* for 'astronomy' appears in *DCMS* ch. 27, 86,16, as pointed out by Hutchinson and Johnson in their commentary on that chapter, and *astronomein*, 'to pursue astronomy', as Merlan pointed out, pp. 150–1, at the end of the present chapter in 74,4, a possible sign that these represent a comment by Iamblichus, and at any rate not a quotation from Aristotle. RRKS

163 Scholium: '"well-founded": Thucydides says the same in Book 1'.

164 The kinship of mathematical studies to philosophy (73,16–17) is next used (73,17–74,5) to account for the Pythagoreans' interest in mathematics and for their coupling it with the study of the cosmos, for example in astronomy, which they described in terms of commensurate numbers and ratios, i.e. ones with a common measure, predicting celestial circuits on the basis of mathematics as a principle, with theological implications. Merlan suggests (p. 150) that this reference to the Pythagoreans may be in Iamblichus' own words to close the chapter, rather than something he is quoting, and points to *astronomein* at 74,4 as a non-Aristotelian term, even though the Aristotelian term for 'astronomy', *astrologikê*, had occurred earlier in that part of the chapter. RRKS

165 On commensurate numbers, see the note on ch. 9, 41,13–14 above. RRKS

166 *astronomein*, the non-Aristotelian term cited by Merlan. RRKS

167 The expressions 'convert the reasoning faculty' (*periagein dianoian*) and 'transport' (*diaporthmeuein*), were prepared for in ch. 1. They are not Aristotelian terms. The first stems from Plato: DSH and MRJ find *periagôgê*, *periagein* of the mind in Plato, *Statesman* 270A, *Republic* 518D4, E4, 521C6, 533D3, and JMD, in

his note on 10,19, finds the second term, *diaporthmeuein*, in Plato, *Symposium* 202E. The reference to pure forms (*eidê*) and reason-principles (*logoi*) is also of Platonic inspiration and is seen by DSH/MRJ as Neopythagorean from the later time when Pythagoreanism was linked with Platonism. RRKS

168 Iamblichus calls Pythagoras by name here at 74,23, as in ch. 23, 70,1–3, and gives a very individualistic account, whereas Aristotle had normally spoken of the Pythagoreans without trying to decide what had been inaugurated by Pythagoras, ch. 23, 73,18–19; ch. 25, 78,8; ch. 24, 75,5; ch .27, 87,16–17. So Burkert, *Lore and Science*, pp. 29–30. From DSH

169 Geometrical application in Euclid is the application to a straight line of areas equal, exceeding, or falling short of, a given area. See T. Heath, *A History of Greek Mathematics* (Oxford: Clarendon Press 1921), vol. 1, pp. 150 ff. The second problem is that of trying to construct a square equal in area to a given circle. RRKS

170 Cf. the different treatment for different students in ch. 18, which fits also with the division of students into Mathematicals and Listeners in ch. 25. RRKS

171 This is presumably a reference forward to the later books of the 'Pythagorean Sequence', and specifically Books 5–9, on Physics, Ethics, Music, Geometry, and Astronomy respectively (all, alas, now lost).

172 Acousmatics formed an outer circle of devotees of Pythagoras' sayings; Mathematicals developed his mathematical physics.

173 Demonstrations are formal proofs. Those in mathematics are likely to be more complex than Aristotle's syllogisms confined to two premises and a conclusion. See the notes on analysis and synthesis at 59,7 and 65,15. RRKS

174 After a very brief editorial opening by Iamblichus, lines 76,18–78,8 of this chapter overlap with the first work in Iamblichus' Pythagorean series, *On the Pythagorean Life*, ch. 18, §§ 81 and 87–89. W. Burkert, *Lore and Science in Ancient Pythagoreanism* (Cambridge, MA: Harvard University Press, 1972), p.193, n. 8, says that §§ 82–87 of Iamblichus *On the Pythagorean Life* are drawn in turn from Aristotle's own account of the two types of Pythagorean, and he refers on pp. 28–9 to Aristotle's fragmentary work, originally in one or two books, *On the Pythagoreans*. He does not consider whether Aristotle's *Exhortation* also lies in the background. From DSH.

175 Iamblichus contradicts this in one of his two reports in *On the Pythagorean Life*, saying at ch. 81, 46,26 ff. that the Listeners (*akousmatikoi*) were accused by the others of having been inaugurated later by Hippasus, not by Pythagoras. But ch. 87, 51,12 ff. keeps the current version. Perhaps Iamblichus was tempted at the one point to cite the version unfavourable to the Listeners, because his own sympathies lay with the Mathematicals. From DSH. Aristotle was able to avoid

controversy about what Pythagoras invented, because he regularly refers not to Pythagoras as an individual, but to the Pythagoreans, as noted by Burkert, *Lore and Science*, pp. 29–30, and in the note on ch. 24, 74,23 above, unlike Iamblichus, whose individualistic treatment of Pythagoras was exemplified in the previous chapter.

176 Croton, where Pythagoras founded his school. RRKS

177 Polycrates (sixth century BCE) was a Greek ruler who seized power in the island of Samos. SG

178 A correction for hexagons (six-sided), which cannot make a dodecahedron (twelve-sided). From DSH. In *On the Pythagorean Life* 18, Hippasus is said to have used twelve pentagons to accomplish the same feat instead. Cf. also Plato, *Phaedo* 110B. SG

179 Theodore went from Cyrene in Africa to Athens in the fifth century BCE and is credited in Plato's *Theaetetus* with discovery of which square roots of numbers up to 17 were irrational, and by tradition with the 'square root spiral' composed of adjacent right-angled triangles. RRKS

180 Hippocrates of Chios, author in the fifth century BCE of the lost but earliest recorded *Elements of Geometry*, researched squaring lines and constructing a cube of double volume. RRKS

181 The remark 'Geometry was called "inquiry" (*historia*) by Pythagoras' (25,39) fits awkwardly with the preceding account of how mathematics sprang from a desire for monetary gain, unless one is to give the Greek word *historia* the sense of 'seeking out [*sc*. financial advantage]', as F. Romano suggests in *Giamblico*, *Summa Pitagorica* in his note at this point. SG

182 i.e. that of the Mathematicals and that of the Listeners.

183 78,13–20. How strong a sense should be given to the idea that for early Pythagoreans numbers, the geometrical forms of things, and definitions are 'the causes of existents and their principles' and the causes of all things, and that anyone must look to these if they wish to view the nature of existents – more literally, how existing things are (or, one might say, how they have their being)? These mathematical objects must be ultimate explanatory factors, and the expression 'all things were made clear by these' confirms that they have a fundamental role as *explanatory*. But the wording here on its own does not tell us whether the status of mathematical entities as *beings* is also fundamental.

It is rather Speusippus, Plato's nephew, in his Academy, as discussed in chs 3 and 4 above, who first speaks of such beings as a One, which may be above being, and a plurality principle, which sound like higher entities that produce the unity and plurality of the monads which make up numbers, while in ch. 9 Plato's immediate successors in the Academy, Speusippus again and Xenocrates, treat the

soul as a mathematical entity, and Iamblichus welcomes this and combines together their definitions of soul. But by Iamblichus' time, Neoplatonism, starting with Plotinus in the third century CE, had taken up the hierarchical role of the One, although he did not conceive it mathematically, and a Neopythagorean, Moderatus of Gades, in the first century CE may have welcomed a mathematical definition of soul as a harmony of consisting of ratios (see the note on *DCMS* ch. 9, 40,17). Iamblichus refers further in *DCMS* ch. 31, 94,22–6 to unnamed Pythagoreans making use of numbers, harmonics, figures, and other mathematical entities to throw light on the soul. RRKS

184 Iamblichus' closing comment. RRKS

185 Much of this chapter (79,5–81,7 and 86,6–22, Festa) was argued by W. Jaeger, *Aristoteles*, 1923, 2nd edn 1955, 60–80 (in the English edition, *Aristotle, Fundamentals of his Development*, 1962, ch. 4, 'The Protrepticus'), to be taken from Aristotle's *Exhortation to Philosophy* or *Protrepticus*, and is included as frr. 52–53 in Rose's edition, 5b and 8 in Walzer's. Certainly the style is no longer that of Iamblichus.

186 D. S. Hutchinson and Monte Ransome Johnson stress a fresh point at www.protrepticus.info/protr2017x20.pdf, in their 2017 commentary on *DCMS* ch. 26. Not only does Iamblichus' *Exhortation* quote Aristotle's *Exhortation to Philosophy*, but so also does ch. 26 of Iamblichus' *DCMS* in parallel to ch. 6 of Iamblichus' *Exhortation*. The *DCMS* subject is objections to theoretical mathematics (from *DCMS* 80,5–81,4) and replies (from 81,5–84,20). DSH and MRJ argue that both of these two works of Iamblichus are in turn quoting from the same earlier source identified by Jaeger and Merlan (Aristotle's *Exhortation*) and that Aristotle had there put objections to theoretical mathematics into the mouth of Isocrates and had put replies into his own mouth. DSH and MRJ had already used *DCMS* in reconstructing lost parts of Aristotle's *Exhortation*. They warn that Iamblichus' borrowings from Aristotle's *Exhortation* differ in these two works of his. Thus the particular *objections* given here by Iamblichus in *DCMS* do not appear in Iamblichus' *Exhortation*, whereas the *replies* given here in *DCMS* up to 83,2 can be seen as a condensed version of those in Iamblichus' *Exhortation*, ch. 6, from 37,26 to the end of that chapter. However, the remaining replies here from 83,2 to 84,20 quote Aristotle more extensively and do not appear in Iamblichus' *Exhortation*. There are also further passages in *DCMS* ch 26, they argue, which contain overlooked parallels with Iamblichus' *Exhortation* ch. 6, and these too may reflect Aristotle's earlier *Exhortation*. Thus (quite apart from lesser parallels between *DCMS* 81,5–83,5 and Iamblichus *Exhortation* 37,26–41.4) *DCMS* ch. 26, 82,17–83,2, is almost word for word identical with Iamblichus *Exhortation* ch. 6, 40,12–41,2, and so equally is 81,20–24 of his *DCMS* ch. 26 with 38,22–39,4

of ch. 6 of his *Exhortation*. But neither Rose in 1886 nor Walzer saw the passage as a possible fragment from Aristotle's earlier *Exhortation*. Yet both *DCMS* and *Exhortation* preserve a telltale use of verbs in the first person, 'I think' (82,15), 'it seems to me' (82,20–21), which could reflect the direct speech of speakers which DSH and MRJ attribute to the original dialogue form of Aristotle's *Exhortation*. And *DCMS* preserves one extra first person use, 'I was convinced', at 82,17 which is not in the first person in Iamblichus' *Exhortation*. Further, *DCMS* ch. 26, 83,24–84,20 is useful for having preserved a complete argument relevant to the reconstruction of Aristotle's *Exhortation*, which is not preserved in Iamblichus' *Exhortation*. Finally, Proclus in Book 1, ch. 9, 25,15–29,13, of his *Commentary on Euclid 1* mentions ideas from *DCMS* ch. 26, and attributes two of the arguments to Aristotle, in one case at 26,13, adding that the argument from the beauty of mathematical objects was trying to 'persuade' us (*peithein*), a verb which is reminiscent of Aristotle's *Exhortation to Philosophy*. From DSH/MRJ

187 Iamblichus now concludes the historical introduction. RRKS

188 'Both the many and the sophisticated [. . .] take living well and good practice to be the same thing as to be *eudaimon*', Aristotle, *Nicomachean Ethics* 1095a18–20.

189 80,1–5, the first of the quoted objections to mathematics: it has no practical *telos* (goal). Surveyors, musicians, and navigators are more useful. This is an objection to mathematical studies such as those of Plato's Academy, as reflected in Plato, *Republic* Book 7. The objection continues down to 81,4 and in light of its similarity to the objections in Isocrates' *Antidosis* 262, the reconstruction by Hutchinson and Johnson of Aristotle's *Exhortation* takes it to have there been put by Aristotle into the mouth of Isocrates. From DSH/MRJ

190 80,5–81,4. The objection uses a distinction elaborated in Aristotle's later work, the *Posterior Analytics*, at 1.7, 75b12–20; 1.13, 78b32–79a7; and 1.27, 87a31–36. There optics is subordinated to geometry, harmonics to arithmetic, mechanics to solid geometry, star-tracking to astronomy, and nautical astronomy to mathematical astronomy. The present objection in Iamblichus *DCMS* regards the navigational astronomer as more useful than the mathematical astronomer, the practically trained musician than the expert in harmonics, and the land-surveyor than the geometer. From DSH/MRJ

191 The philosopher, like the pure mathematician, is here tarred with the brush of being a useless spectator. The philosopher/astronomer, Thales, was caricatured by Iamblichus in his *Exhortation* ch. 14 as the absent-minded philosopher, on the basis of the story in Plato, *Theaetetus* 174A that he was laughed at by a witty slave girl for falling down a well while contemplating the stars.

192 As in *DCMS* ch. 23, 72,18–19, the term *astrologia* or *astrologikê*, is the one used by Aristotle for astronomy. Regarding the two cases in *DCMS*, where the word

used for astronomy is instead *astronomia* or *astronomein*, in ch. 23, 74,4, and ch. 27, 86,16, Merlan points out on p. 150 of his *Platonism and Neoplatonism*, 1960, that the second term is not an Aristotelian usage and it is Iamblichus who is making a comment at that point. Both terms are regarded by Hutchinson and Johnson as a sign of Iamblichus' intervention. RRKS

193 The sentence from 81,5–7 is Iamblichus' editorial comment marking the transition to the reply in defence of mathematical sciences. Its abruptness is taken by Hutchinson and Johnson to reflect the dialogue form of the postulated source (*Aristotle's Exhortation*) of the objections and replies. RRKS

194 This sentence is marked as Iamblichus' editorial comment by DSH/MRJ .

195 This sentence is marked as Iamblichus' editorial comment by DSH/MRJ.

196 At 82,14–17, Iamblichus, having finished with defending the utility and greatness of mathematical philosophy, introduces the new topic, going down to 83,2 of the easiness of acquiring it. From DSH/MRJ

197 Cf. below 83,13–21, and the parallel with Iamblichus, *Exhortation* ch. 6, 40, 15–20. From DSH/MRJ

198 83,2–5: Iamblichus' summary to date. From DSH/MRJ

199 Zeus flooded the earth in anger; Deucalion, the son of Prometheus, and Pyrrha alone survived to re-establish the human race. Cleinias, in Plato's *Laws* at 677A, says that such stories are very plausible. JMD. But Iamblichus is likely to be thinking of Aristotle, who took the stories more literally, and spoke of recurrent floods, *Physics* 4.13, 222a23, 26, where he uses the word 'cataclysm' (*kataklusmos*) here repeated by Iamblichus. He thought that knowledge would have to be recovered possibly an infinite number of times, e.g. *Meteorology* 339b29, *On the Heavens* 270b19, *Politics* 1329b25, *Metaphysics* 1074b10. When Philoponus, like Iamblichus before him, wrote a commentary on Nicomachus' *Introduction to Arithmetic*, (1.1.5–49), he told a story about the rise of human arts after a cataclysm, referring to Aristotle, and to the flood of Deucalion. From DSH/MRJ

200 'Devote their thought'. This sense would be better provided by reading *phrontizein* in 83,12 and treating *philosophein* (philosophizing) as having crept in from its mention below as a later development. This would fit with the reference to *phrontis* (concern) in the parallel passage of Proclus' commentary on Euclid's Book 1, at 29,1–2. From DSH/MRJ

201 'In so short a time': cf. above 82,17–22 and the parallel with Iamblichus, *Exhortation* ch. 6, 40,15–20. Cicero, *Tusculan Disputations* 3.69, reports Aristotle as rebuking philosophers of old for saying that thanks to their genius philosophy had reached perfection, but as conceding that because of the great advance made in a few years (presumably in Plato's Academy), philosophy would be complete. From DSH/MRJ

202 Cf. Iamblichus, *Exhortation*, ch. 9, 51,16–23 Pistelli. The principle is used by Aristotle in his biological account of the development of an organism, e.g. *On Parts of Animals* (*PA*) 1.1, 640a19–26, 641b23–642a1, 2.1, 646a25–7, *On the Generation of Animals* 2.1, 734a16–32, and 2.6 throughout. From DSH/MRJ

203 At 83,24–5 the case for exactness and beauty is not made in what follows down to 84,7, although DSH/MRJ argue that the case for beauty may have been found somewhere in Aristotle's *Exhortation* as source. For Proclus' *Commentary on Book 1 of Euclid's Elements* Prologue 1, ch. 9, 26,10–27,10 locates an argument for beauty in mathematical objects among 'arguments by which Aristotle tries to persuade us'. DSH/MRJ take the omission and other features as one sign that Iamblichus is not quoting, so much as freely adapting, his source. As other signs they point to the phrase at 84,3–4, 'what frees us from our bodily nature' which I agree suggests Iamblichus' insertion, but because he believes, while Aristotle disbelieves, in our being able to outlast our bodies. Further, they point out that Aristotle's word for 'bodily' is *sômatoeidês*, not *sômatôdês*. From DSH/MRJ

204 84,7–20: DSH/MRJ point out that the utility of mathematics, which has already been discussed in ch. 26 of *DCMS*, will be picked up in Iamblichus' related text, the *Exhortation*, in its next chapter, ch. 7. The disorder in presentation suggests that Iamblichus is not quoting. Moreover, the claim at 84,11–14 that the contributions (*erga*) of mathematics are worth less than the purification of the human soul, must be Iamblichus' own comment and is far removed from Aristotle's sentiment. At 84,13 the turning (*periagôgê*, cf. ch. 2, 10,2 and ch. 24, 74,11, *periagein*) as applied to *intellect* is also not an Aristotelian expression. Iamblichus seems to be giving his own reasons for valuing mathematics, not previously offered in the chapter. From DSH/MRJ

205 Unusually, there is no introductory comment by Iamblichus in *DCMS* ch. 27, but the chapter corresponds pretty closely for about a quarter of its length with the opening of Aristotle's *On the Parts of Animals* from 1.1, 639a1–b6, starting with Aristotle's second sentence there. The obvious conclusion might seem to be that Iamblichus has been copying from his predecessor Aristotle's *On the Parts of Animals*. But DSH and MRJ in their 2017 online 'info/evidence' publication argue instead that Aristotle's *PA* and this section of Iamblichus' *DCMS* are alike adaptations of a lost section of Aristotle's *Exhortation to Philosophy*.

For one thing, it would be an unlikely project for Iamblichus to undertake, if he was trying to explain the methodology of the exact science of mathematics by *starting* from the methodology of the least mathematical of the sciences, zoology. By contrast the recognized fragments of Aristotle's *Exhortation to Philosophy* have a few mathematical examples, but could easily be converted to a source for the support of different subjects: both zoology in Aristotle's *On Parts of Animals*,

and later theoretical mathematics in Iamblichus' *DCMS*. One can see how Iamblichus could have converted the *PA* text to apply to mathematics, but not why he would have wanted to start his treatment of mathematics from there.

The conversion of *PA* would have involved cutting out the first sentence of *PA* 1.1, 639a1–b6 which introduces the need for the educated person who, without knowing the truth about a scientific specialism, can judge the proofs offered. The first sentence of *PA* 1.1 says that this is needed equally in humble and more honourable subjects. *DCMS* takes up the need for the educated judge, but omits the reference to more humble subjects because it is not concerned with zoology. But on the hypothesis that Iamblichus is following a lost part of Aristotle's *Exhortation*, no conversion would have been needed, because the known fragments of Aristotle's *Exhortation* are also not trying to accommodate zoology.

Whether Iamblichus was, unexpectedly, copying Aristotle's *PA*, or a lost part of Aristotle's *Exhortation*, either he would have deleted sections of *PA*, or Aristotle would have adjusted for *PA* parts of his *Exhortation*.

DCMS 84,25–85,3 is about mathematics, so that and the reference to mathematical education at 85,25–86,2 would either have been added by Iamblichus to *DCMS* or deleted by Aristotle in adapting his *Exhortation* to *PA*. Similarly, with *PA*'s references to the study of nature (639a12), or of each substance, e.g. human, lion, or ox (639a17), such common properties as sleep, respiration, growth, decay, death (639a20–1), the different forms of animal motion, (639b1–3). Either Iamblichus had to delete them, or Aristotle added them on the hypothesis that he adapted a lost part of his *Exhortation* to *PA*.

A second consideration, besides the unlikelihood of Iamblichus arguing from zoology to mathematics, is the presence of an exhortation to study zoology in Aristotle's *PA* at 1.5. Aristotle had earlier on in his *Exhortation* argued that beauty exists in the mathematical sciences. Here in *PA* 1.5, he takes it for granted that the objects at least of astronomy are divine and worthy, but he says he has already discussed the philosophy of divine things. So now it remains to speak of animals, leaving out none, whether more or less worthy. As was argued in Iamblichus' *DCMS* ch. 26 about mathematical objects, so in *PA* 1.5, 644b17 Aristotle argues that the study of animals is possible and easy (644b17; 30), and he opposes the claims of those who consider such study disagreeable to the senses (645a7) and who are childishly disgusted at the study of the less worthy animals (645a15), since in all natural things there is something wonderful. The philosopher Heraclitus said, 'There are gods even in the kitchen' (A9 in Diels-Kranz, *Fragmente der Vorsokratiker*, from Aristotle, *Parts of Animals* 1.5, 645a15–23), so we should not grimace at approaching research on each of the animal kinds, since there is something natural and beautiful in them all. This is because

purpose is found especially in the works of nature and the function for which [animal parts] have been formed has its place in what is beautiful. Those who feel disgust are looking at animal matter instead of at the form of the whole organism.

Iamblichus similarly argues in his *Exhortation to Philosophy* ch. 9, 50,16–51,4, that anything that comes into being, or has come into being by nature, comes into being, or has come into being beautifully. This is so, if for what is in accordance with nature, coming into being serves a purpose. You could see this from each of the parts of our own bodies, for example the eyelid, which helps the eye. And surely animals are things that have come into being by nature, all of them – or at least the best and worthiest, since it makes no difference if anyone thinks that most animals have come into being unnaturally through some corruption or bad condition (*phthora*, *mokhthêria*). Rather, their parts have come into being to serve a function and by nature. Thus for a second time, we see some of the same ideas being canvassed in Aristotle's *PA* and in a work of Iamblichus that has for some time been seen as in parts quoting Aristotle's *Exhortation*. From DSH/MRJ

206 Proclus, *On Euclid's Elements 1*. 32,23–24, cites this as a saying of Aristotle, and it comes from Aristotle, *Nicomachean Ethics* 1.3, 1094b27–1095a2. From DSH/MRJ

207 Aristotle says, 'underlying matter' (i.e. subject-matter) at *Nicomachean Ethics* 1.3, 1094b12. From DSH/MRJ

208 Proclus, *On Euclid's Elements 1*, on his next Page, 33,24–6, again attributes such a saying to Aristotle, and indeed it is a version of what Aristotle says in the same passage at *Nicomachean Ethics* 1.3, 1094b 25–7 in his explanation of why statements in ethics hold true only for the most part. From DSH/MRJ. Their view is that Aristotle draws on his earlier *Exhortation* in later works, including both *Nicomachean Ethics* and *On Parts of Animals*.

209 This African tree was used for the manufacture of the *aulos*, which presumably required great accuracy if the instrument was to be in tune.

210 The harmonic ratios of elastic string lengths, and the geometrical patterns made by moving stars are less exact than pure arithmetic and geometry. But cf. the argument in *DCMS* ch. 26 above, 80,5–81,1 making all of these less useful than playing musical instruments or tracking the stars.

211 86,16: *astronomia* is not Aristotle's term (*astrologikê*) for astronomy, and may have been substituted by Iamblichus, as with *astronomein* in ch. 23 at 74,4. RRKS

212 87,23–29: the idea that there are principles superior to those of mathematics has not been acknowledged by Iamblichus, and so this passage may still be reporting views of Aristotle. On the reconstruction of Iamblichus *DCMS* ch. 26 and Iamblichus *Exhortation* as both quoting from Aristotle's *Exhortation*, Aristotle will have chosen not mathematics in particular, but more generally philosophy (Iamblichus *DCMS* 83,20–22) and intelligence and theoretical knowledge

(Iamblichus *Exhortation* 43.22; 43,22–24) as more valuable than mathematics taken on its own. From DSH/MRJ

213 Iamblichus' closing editorial comment to ch. 27 may be no longer than this. RRKS

214 The opening of ch. 28, at 88,12–21 appears to continue 88,2–10 in the preceding chapter. That had concluded, probably following Aristotle, that mathematical principles are not the principles of everything, but only of some things. The new chapter's opening, perhaps still drawing on Aristotle, now concludes that we must on the one hand distinguish mathematical research and demonstration from other kinds, and on the other hand see how the kinds are connected. From DSH

215 88,21–26: with the word 'Now' (*men oun*) at 88,21 Iamblichus takes up not the first theme, the *distinctness* of mathematics, but, still following Aristotle, the second theme at 88,18–26 of its *interconnexion* with other studies. And the distinctness is even blurred when Iamblichus goes on to report some Pythagorean source at 88,26–30 which makes the interconnexion closer still. Iamblichus' reminder of the first theme, the need for distinctness, is postponed until after that. See the note on 88,30–89,2 below. RRKS

216 88,26–30: This is where Iamblichus gives from 88,12–21 the very different Pythagorean source which connects things up 'more closely' than Aristotle. RRKS

217 88,30–89,2: with 'However', Iamblichus gives his postponed reminder that we need to distinguish mathematics proper from the Pythagoreans' mathematical treatment of intelligible forms, theology, and ethics. RRKS

218 89,2: Iamblichus emphasizes the need not to confuse the mathematical realm with others, so that he accepts Aristotle on the need to distinguish mathematics proper. But he then reminds us at 89,5 of his rejection in ch. 8, 34,10 above of Aristotle's downgrading of the objects of mathematics as a product merely of our minds *abstracting* numbers from sets of material objects and geometrical entities from the inexact shapes, edges, and corners of material objects. The relevance, though not obvious, is presumably that Aristotle, in his theory of abstraction, after all fails to distinguish the mathematical sphere sharply enough from perceptible bodies which we subject to the mental process of abstraction. In ch. 32 below, at 93,19, where such abstraction is accepted, Iamblichus is presumably quoting an Aristotelian position. RRKS

219 90, 8–9: on *kritêrion*, see the note on the chapter heading to ch. 8 at its second occurrence. RRKS

220 90,8–12: besides the three announced facilities, syllogisms, divisions, and definitions, these lines add two further autonomous sources of development to mathematics: criteria for approving (or testing) definitions for oneself and

demonstrations. The point being made is that mathematics does not depend on philosophical dialectic, as Plato thought, but has its own resources. From DSH

221 What follows this editorial comment of Iamblichus looks as if it is reporting a source. From DSH

222 Iamblichus appears to resume his editorial comment. From DSH

223 Up to here looks to me like an editorial introduction. RRKS

224 The mathematics of number, harmonics, and geometry are used by the Pythagoreans even for psychology. We have already seen in a note on ch. 9 that a Neopythagorean of the first century CE, Moderatus of Gades, may have defined the soul as a harmony consisting of ratios, and that Aristotle had attacked a definition of soul as a self-changing number (see the note on *DCMS* ch. 9, 40,17). RRKS

225 Iamblichus' final comment on the chapter is confined to this short sentence. From DSH

226 DSH conjectures that this may be taken from a speech in favour of mathematical philosophy in Aristotle's *Exhortation*. If so, my first inclination is to suggest that it will have been the speech Aristotle puts into his *own* mouth at the time when he was still a devotee of the interpretation of mathematics in Plato's Academy. At any rate, Iamblichus, or the speaker he cites, allows both for a huge range of methods for applying mathematics to the perceptible world, including first Aristotle's eventual favourite *abstraction*, and juxtaposition (*epharmogê*) a term whose cognate verb is much used by Aristotle. But then the speaker goes on again to take a view not universally accepted, that there is something *greater* and *higher* than the mathematical, and that because of the kinship of mathematical objects with that, an inquiry that starts as mathematical may sometimes be drawn up higher to the essences (*ousiai*) of things. Such a view would have been congenial to the belief of Plato's Academy in the world of Platonic Forms as the highest and to Aristotle in his early days. RRKS

227 It was Plato, in his *Timaeus*, who composed the four elements, earth, air, fire, and water, out of four of the five regular solids, that is, the five three-dimensional closed geometrical figures which have all their faces the same. But others too had to discuss the mathematical ratios in which elements were mixed to produce more familiar perceptible bodies. RRKS

228 'Abstraction' represents Aristotle's eventual view, when he started downgrading mathematical objects as the products of our minds. Both of Aristotle's views were known to Iamblichus. See the notes on ch. 8, 34,10 and ch. 28, 89,5 above, where Iamblichus rejects Aristotle's account of mathematical objects as the product of abstraction by our minds. But in a tour de force worthy of Aristotle himself, Iamblichus, or as I think the speaker he cites, offers many other modes of

application of mathematics to the perceptible world. The next is *juxtaposition* (*epharmogê*), used much later by Simplicius, *On Aristotle On the Heavens* 184,21, to attack Philoponus by saying that one tests the truth of a judgement by applying *truth* to it and the straightness of a line by applying a *straight* edge to it, and comparing to detect deviation or fit. The next mode of application of mathematics to the perceptible is *completion,* to make the imperfect perceptible closer to the mathematical. The next is *assimilation*: treating the perceptible as like the perfectly equal or commensurate mathematical model. The next is *participation*, or seeing how the defining reason-principles of the perceptible participate in reason-principles of pure mathematics. Next, the perceptible may merely *reflect* the mathematical obscurely. Or we may have to notice that an indivisible mathematical form is shared out among *divided* species or individuals. Alternatively, we may have to *compare* the forms in perceptibles with pure mathematical forms by surveying them together. Again, we may have to look for *explanation from prior mathematical principles* by surveying them together with perceptibles to see how the latter might be derived from the former. By allowing for so much variation of methodology, the speaker seeks to make it more plausible that mathematics is extensively applicable to the perceptible world. RRKS

229 See the note on 93,12 for a greater and higher realm than mathematics to which mathematics is merely akin as suiting Plato's Academy and Aristotle in his early days. RRKS

230 Iamblichus' rather spare editorial remark after a rather breathtaking quotation. RRKS

231 Reading *touto to* at the beginning of 94,26. The text reads *tou to*.

232 95,1: Iamblichus here puts a 'crown' on the discussion of the common genus of mathematics. The intermediate nature of mathematics, between intelligibles and perceptibles was stressed in ch. 1 and made clearer a little later in ch. 8 by its position in the divided line. Now it provides the genus and common unity. As regards the proposal for dichotomizing (cutting in two), in ch. 1 at 9,4–11, Iamblichus proposed after seeking a single essence of mathematics, to see whether it could be dichotomized into two principles, and then whether it had a finite number of kinds, following a technique which JMD's note there points out is recommended by Socrates in Plato's *Philebus* 16D. RRKS

233 DSH has suggested that the whole chapter may be Iamblichus without quotation, since he sees no clear sign of quotation and in ch. 1 at 9,4–11 it was Iamblichus himself who proposed to follow the progress from one by dichotomy to two and then to a definite number, although that would not itself prevent Iamblichus basing his view on a quotation. From DSH

234 In 95,5–14 Iamblichus offers some real suggestions about unifying relationships within mathematics – he might almost have been summarizing a source. But as regards dichotomizing mathematics, as proposed in ch. 1 at 9,4–11, after seeking its single essence, the vagueness of this long-awaited application of dichotomy reaches a climax in 95,24–8 with the appeal to the geometrical relations, succession, continuity, and contiguity, to which Aristotle had given very precise geometrical definitions in his *Physics* 5.3 in terms of particular relationships between surfaces. It is not clear from Iamblichus' hints how anything so precise could be applied to kinds of mathematical object, and as if aware that some quite different relationship might be needed, Iamblichus adds an extra term *sunekheia* to cover any other geometrical *connexion* that may turn out to be relevant. But this term is cognate with Aristotle's term for continuity, so that it is still not clear what difference Iamblichus intends. On other things too he has not decided, as at 95,21–2, when he speaks of 'making distinctions quantitatively or in some other way'. RRKS

235 'acquired knowledge' and 'mathematical' – cannot be adequately represented. Mathematics is so called by a narrowing down of meaning from the original 'acquired knowledge in general'.

236 Surely a reference, whether direct or indirect, to the famous passage of the *Iliad* (5.127), where Athena scatters the fog (*akhlus*) from the eyes of Diomedes, so that he may discern both gods and men in the fighting.

237 96,28: *eidôla*, representations. The Pythagoreans are said to have seen mathematical philosophy as discovering representations, not causes, of the natural world, changeless representations providing a model for stable productions (97,6–8), and a source of intellectual understanding of that natural world, (97,12). RRKS

238 Iamblichus' closing editorial comment. I would guess that the well formulated preceding lines are based on sources. RRKS

239 The order within each chapter is determined to some considerable extent by the sources cited, but between chapters it may be largely of Iamblichus' choosing.

240 The single conspectus or review of the whole book occupies in the Greek one single giant sentence of thirty-two lines, the second of the chapter's three sentences, from 97,23–98,28. The first and last sentences look like Iamblichus' editorial comment. From DSH

241 Chapter 2 (all cross-references from DSH).

242 Chapters 3 and 4.

243 Chapter 5.

244 Chapters 6 and 7.

245 Chapter 8.

246 Not a clear reference to either ch. 9 or ch. 10, unless, RRKS, 'determinate' refers back to ch. 9, 41,7.
247 Chapter 11.
248 Chapters 12 and 13. There appears to be no mention of ch. 14.
249 Chapter 15.
250 Chapter 16.
251 Chapter 17
252 Chapter 18.
253 Chapter 19.
254 Chapter 20.
255 Chapter 21.
256 Chapter 22.
257 Chapter 23.
258 Chapter 24.
259 Chapter 25.
260 Chapter 26.
261 Chapter 27.
262 Chapter 28, and 'it' may refer to 'research' (*zêtêsis*) in the first line of ch. 28. From DSH
263 Chapter 28.
264 Chapter 29.
265 Chapter 30.
266 Chapter 34.

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English–Greek Glossary

Compiled by Sebastian Gertz

ability to judge: *kritikon*
absolute: *kat' heautos*
abstraction: *aphaeresis*, *apostasis*
account: *logos*
accuracy: *akribeia*
accurate argument: *akribologia*
acousmatics: *akousmatikoi*
acquired knowledge: *mathêmata*
acquisition of knowledge: *eidêsis*, *mathêsis*
acquisition: *ktêsis*
action: *prattein*, *praxis*
activate: *energein*, *kinein*
active: *kat'energeian*
activity: *energeia*, *praxis*
actualization: *energeia*
adept at studying: *theôrêtikos*
aggregation: *sunthesis*
agreement: *homologia*
aim: *prothesis*, *telos*
akin: *sungenês*
analogously: *kat'analogian*
analogy: *analogia*
analysis: *analusis*
apparition: *phantasma*
applied: *tekhnikos*
apprehension: *noêsis*
approach: *methodos*
argument: *logos*, *sullogismos*
arithmetic: *arithmêtikê*
art: *tekhnê*
arts and crafts: *tekhnai*
as a matter of terminology: *logôi*
as such: *kat' heautos*
ascent: *anhodos*
assemble: *sunagein*
assimilation: *apeikasia*, *homoiôsis*, *homoiotês*
association: *koinônia*
assurance: *pistis*
astronomical: *astrologikos*
astronomy: *astrologia*, *astronomia*
at the level of theology: *theologikôs*
awareness: *theôria*

bad: *kakos*
bad (n.): *kakon*
barbarian: *barbaros*
beginning: *arkhê*
be a judge: *kritikos*
be a philosopher: *philosophein*
be consequent upon: *parakolouthêma*
be prior to: *prohuparkhein*
be puzzled: *aporein*
be supreme over: *arkhitektonikos*
beautiful: *kalos*
beauty: *kallos*, *kalon*
becoming: *genesis*
being turned around: *metastrophê*
being: *einai*, *on*, *onta*, *ousia*
beings: *onta*
belief: *doxa*

bind: *perihorizein*
blessed by fortune: *eudaimôn*
bodily: *sômatoeidês*
body: *sôma*
bond: *dektikos*
bound: *metron*
branch: *genos*, *meros*
by means of arithmetic: *arithmêtikôs*
by means of geometry: *geômetrikôs*
by means of harmonics: *harmonikôs*

calculation: *logismos*
capacity: *dunamis*
category: *genos*, *katêgoria*
causal principle: *aitia*
cause: *aitia*
certainty: *bebaiotês*
chain: *desmos*, *halusis*
change (n.): *kinesis*
change: *metaballein*
character: *eidos*, *genos*, *hexis*, *kharaktêr*
characteristic: *kharaktêr*
choice: *prohairesis*
choiceworthy: *hairêtos*
circuit: *periphora*
circular motion: *periphora*
city: *polis*
clarity: *energeia*, *katharon*, *katharotês*, *saphêneia*
class of being: *genos*
class: *eidos*
cleansing away: *apokatharsis*
cognate: *sungenês*
cognition: *gnôsis*
cognitive: *gnôristikos*, *gnôstikos*
cognizant: *gnôstikos*
coherence: *homologia*
collect together: *sunagein*
collect: *sunagein*
collection: *sunagôgê*
combination: *sunthesis*
coming to be: *genesis*
commensurate: *summetros*
common: *koinos*
common bond: *sundesmos*
common characteristic: *koinotês*
common features: *koina*
common identity: *koinon*
common nature: *koinotês,*
community: *koinônia*
comparison: *antidiataxis*, *parabolê*
compass: *diastasis*
compatibility: *homologia*
completeness: *teleiotês*
completion: *sumplêrôsis*, *teleiôsis*
complex in form: *polueidos*
complex: *meristos*
composite: *en sunthesei*
composition: *sunagôgê*, *sunthesis*
compounding: *sunthesis*
comprehension: *katalêpsis*
conceive: *noein*
concept: *ennoia*
conception: *doxa*
conceptually: *epinoiai*
concord: *sumphônia*
condition: *hexis*
confidence: *pistis*
conjunction: *allêloukhia*
connect: *sunaptein*
connected with the arts: *tekhnikos*
connection: *koinonia*, *sundesmos*, *sunekheia*
consensus: *homologia*
consequentiality: *akolouthon*
conspectus: *sunopsis*
constitution: *politeia*, *sustasis*
construction: *sustasis*

contemplate: *theôrein*
contemplation: *thea*
contemplative: *theôrêtikos*
continuity: *sunekheia*
contradictory: *enantios*
contribution: *ergon*
control: *katakratein*
conversion: *periagôgê*, *metastrophê*
convert: *metastrephein*, *periagein*
copy: *eikasma*
corporeal: *sômatikos*
correct form: *orthotês*
correct judgement: *krinein*
correct procedure: *orthotês*
correct research: *zêtein*
correctness: *orthotês*
cosmos: *kosmos*
craft: *tekhnê*
creative: *poiêtikos*
creative activity: *genesiourgia*
criterion: *kritêrion*
critical analysis: *krisis*
critical: *kritikos*
cycle: *periodos*

deficiency: *elleipsis*
define: *horizein*
definite quality: *poion*
definite: *hôrismenos*
definition: *dihorismos*, *horismos*, *horistikê*, *horizesthai*, *logos*
definitional: *horistikos*
definitional process: *horistikê*
deity: *theos*
delicacy: *leptotês*
delimited: *peperasmenos*
demonstration: *apodeixis*
desire: *orexis*
destroy: *sunanhairein*
destruction: *phthora*
determinant: *horos*
determinate: *hôrismenos*
determination: *epikrisis*
determine: *diakrinein*
develop syllogisms: *sullogizesthai*
dialectic: *dialektikê*
dichotomy: *dikhotomia*
difference: *diaphora*, *anhomoiotês*, *heteron*, *heterotês*
differentia: *diaphora*
discipline: *askêsis*, *tekhnê*
discovery: *heuresis*
discursive reasoning: *dianoia*
discussion: *logos*
disposition: *diakosmêsis*, *hexis*
dissimilar: *anhomoios*
dissimilarity: *anhomoion*, *anhomoiotês*
distance: *apostasis*
distinction: *diakrisis*, *dihorismos*
distinguish: *aphorizein*, *diakrinein*, *krinein*
distinguishing feature: *horistikê*
discursive reason: *dianoia*
divide: *temnein*
divided: *dihairoumenos*, *meristos*
dividing: *dihairetikê epistêmê*
divine: *theios*, *entheos*
divisible: *meristos*
division: *dihairein*, *horos*, *tomê*
divisive process: *dihairetikê*
doctrine: *didaskalia*, *doxa*
draw together: *sunagein*
due measure: *eumetria*

educate: *paideuein*
educated man: *pepaideumenos*
education: *paideia*
educational system: *paideia*
elaboration: *poikilia*

elegance: *leptotês*
element: *stoikheion*
encompass: *periekhein*
end: *telos*
enmattered: *enhulos*
entity: *on*, *ousia*
equality: *ison*, *isotês*
esoteric: *esôterikos*
essence: *ousia*
essential being: *ousia*
eternal being: *aei on*
eternal: *aidios*
ethical questions: *êthê*
ethical: *peri ta êthê*
ethics: *êthê*
evaluation: *epikrisis*
evil (n.): *kakia*
evil: *kakos*
exactitude: *akribeia*
exactness: *akribeia*
excellence: *aretê*
excess: *huperbolê*
exemplar: *paradeigma*
existence: *ousia*, *hupostasis*, *on*
existents: *onta*
existing things: *onta*
exoteric: *exôterikos*
experience: *empeiria*
experienced: *empeiros*
explanation: *aitia*, *aition*, *logos*
explanatory principle: *aition*
extend: *diateinein*
extension: *diastasis*, *pêlikon*
eye: *omma*

facts: *pragmata*
faculty: *dunamis*
faculty of cognition: *gnôristikon*
fall away: *ekpiptein*
false: *pseudos*
familiarization: *oikeiôsis*
feature: *pathos*
field of study: *theôria*
figure: *skhêma*
filling: *plêrôsis*
fine (n.): *kalon*
fine: *kalos*
fineness: *kallos*, *leptotês*
finger: *daktulos*
firmness: *antitupia*
first principle: *arkhê*
flipping: *peristrophê*
flood: *kataklusmos*
focus: *epibolê*
fog: *akhlus*
form: *eidos*, *idea*, *morphê*
form of being: *ousia*
form of life: *zôê*
formal character: *eidos*
formal proof: *apodeixis*
formal: *eidêtikos*
formulate: *proballein*
free man: *eleutheros*
friendship: *philia*
function: *ergon*, *organon*
fundamental: *arkhikos*, *arkhitektonikos*

gap: *kenôsis*
general: *koinos*
generation: *genesiourgia*
genus: *genos*
geometer: *geômetrês*
geometers: *hoi peri tên geômetrian*
geometric: *geômetrikos*
geometrical application: *parabolê*
geometry: *geômetria*
give an account in accord with principles of natural philosophy: *phusiologein*

give birth: *apotiktein*
goal: *telos*
god: *theos*
good (n.): *agathon*, *kalon*
good: *agathos*
good measurement: *eumetria*
good order: *eutaxia*
goodness: *agathon*
grasp: *katalêpsis*
grossness: *pakhutês*
ground: *aitia*, *arkhê*
group practice: *sunêtheia*
grouping together: *suntaxis*
growth: *auxê*

happiness: *eudaimonein*, *eudaimonia*
harmonics: *harmonikê*
harmonizing: *harmonia*
harmony: *euharmostia*, *harmonia*, *summetria*
have an intuition: *noein*
having many forms: *polueidos*
heaven: *ouranos*
heavenly: *ouranios*
heavenly bodies: *ourania*
heavens: *ouranos*
household: *oikos*
human: *anthrôpinos*
hypothesis: *hupothesis*

identity: *auton*, *tautotês*
illogical: *alogos*
illustration: *paradeigma*
image: *eidolon*, *eikôn*
imaged things: *eikasta*
imagination: *phantasia*
imaging: *eikasia*
immaterial: *ahulos*
impetus: *aphormê*
impress: *episphragizein*
impression: *emphasis*
in due proportion: *summetrôs*
in general: *holos*
in mathematics: *mathêmatikos*
in nature: *phusei*
in number theory: *arithmêtikos*
in order: *taxei*
in potency: *en dunamei*
incapacity: *adunamia*
incorporeal: *asômatos*
incorporeality: *asômatia*
increase: *auxêsis*
indefinable: *aperihoristos*
indeterminacy: *ahoristia*
indeterminate: *ahoristos*
individual (n.): *kat'hen*
indivisible: *ameristos*, *adihairetos*, *atmêtos*
inductive process: *epagôgê*
inequality: *anison*
inform: *parangellein*
inquiry: *episkepsis*, *historia*
insight: *theôrêma*
instruction: *didaskalia*
instrument: *organon*
intellect: *nous*, *noêsis*
intellection: *noesis*, *nous*
intellectual: *noeros*, *theôrêtikos*
intelligence: *noêsis*
intelligible (n.): *noêta*
intelligible: *gnôrimos*, *noeros*, *noêtos*
intelligible realm: *noêta*
intelligible thing: *noêton*
intermediacy: *mesotês*
intermediate: *mesos*
introduce: *sunepipherein*
intuit: *noein*
inventive: *heuretikos*

investigate: *episkepsasthai*, *episkopein*, *skopeisthai*
investigation: *episkepsis*, *theôria*, *zêtêsis*
invisible: *ahoratos*
irrational: *alogos*
irrefutable: *anelenktos*

joining together: *harmonia*
judge: *krinein*
judgement: *krisis*
justice: *dikaiosunê*, *dikê*
juxtaposition: *epharmogê*, *parathesis*

kind: *eidos*, *genos*
kind of object: *genos*
kinship: *sungeneia*, *xungeneia*
knowing: *ginôskein*
knowledge: *gnosis*, *eidêsis*, *episteme*, *ginôskein*, *mathêsis*, *philosophia*, *phronêsis*

ladder: *klima*
largeness: *mega*
lead over: *diatribazein*
learn: *manthanein*
learning: *mathêsis*
legacy: *moira*
level of being: *hupostasis*, *on*, *ousia*
level of existence: *ousia*
level of knowledge: *gnôsis*
level of reality: *on*
level of understanding: *gnôsis*
liberal education: *paideia*
liberation: *lusis*
life: *zôê*
liken: *apeikazein*
likeness: *homoiotês*
limit (n): *peras*
limited (n): *peperasmenon*
limited: *peperasmenos*
limiting principle: *perainon*
line: *grammê*
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